Zero-Trust Post-quantum Cryptography Implementation Using Category Theory

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Abstract

This paper blends post-quantum cryptography (PQC) and zero trust architecture (ZTA) to secure the access for AI models, formalized through the abstract mathematical lens of category theory. In this work, lattice-based PQC primitives are assigned ZTA components that include micro-segmentation and context-aware authentication, leading to a visual compositional framework that describes cryptographic workflows as morphisms and trust policies as functors, showing how category theory allows for fine-grained policies and adaptive trust. This quantum-resistant algorithm viewing perspective offers an ease for protection against adversarial AI threats. The paper uses a concrete implementation to attest to the effectiveness of the theoretical contribution, rendering it a crypto-agile transition using categorical proofs for AI security .

1 Introduction

Category theory gives a framework to capture cryptographic primitives and their interactions, formalized as morphisms and functors to maintain algebraic consistency. Cryptographic protocols are themselves compositions of operations described within this theory as commutative diagrams and universal properties allowing to abstract away from computation details focusing on invariant structure rather than calculation. This frameworks stores probabilistic effects using the monads to distinguish between deterministic logic and randomness, and utilizing functors and natural transformations to encode parameterized schemes, whereas set theory treats objects as unstructured collections but that does not provide relationship information, for instance, the system public matrices relate to secret keys for a problem such as Learning With Errors (LWE).
Nevertheless, the encryption based on this LWE Engel expansions can be ameliorated by deterministic randomness, and provide structure from a seed. This helps in security proofs while efficient sampling for the generation of large matrices directly from a seed saves on costs in communication and storage. Moreover, Engel coefficients are non-decreasing and chaotic under shuffling, structured yet unpredictable, therefore, this nonlinear recurrence sidesteps any linear dependencies that could be targeted in a reduction attack.

2 Related work

This new area combines post-quantum cryptography (PQC) with category theory and zero trust architectures (ZTA) to secure AI models and touches on advances in all three areas: cryptography, mathematical frameworks, and cybersecurity paradigms. At this point, we retrace some of the most important contributions, including an urgent PQC adoption need stressed by NIST-defined quantum-resistant algorithms such as ML-KEM (FIPS-203) and ML-DSA (FIPS-204) [12]. These lattice-based cryptographic tools are meant to nullify attacks from quantum computers with respect to weaknesses leveraged from classical encryption methods. Recently, efforts were demonstrated in a practical approach to PQC deployment within the communications security of a one-time use proofing system by enabling an access model forming the foundation of quantum-safe Zero Trust Network Access (ZTNA) [13]. While most present work on algorithms for PQC has focused on performance analysis, the formalization of such algorithms in the structure of category theory has not yet received the required attention [14]. Consequently, we have here an opportunity to showcase how category theory could model these abstract relations and compositional structures for bringing into one picture the modular components of PQC and the larger systems that such modules can compose among themselves. Zero Trust principles explicit verification, least-privilege access, and continuous monitoring are the key elements in securing AI systems from adversarial attacks, data poisoning, and model inversion [15]. For instance, granular access control systems and micro-segmentation in ZTA reduce risks involved with the AI black-box nature so that only authenticated entities can engage with sensitive models. Another recent work emphasizes the importance of encryption, behavioral analytics, and context-aware policies in securing the entire workflow of AI. The latest proposals suggest Information-Theoretic Security (ITS) protocols for decentralized networks using multi-party computation and ZTA, augmented by blockchain technology, with the intention of addressing ransomware threats [16]. Thus, relying on category theory, the compositional structures can represent the wire-tap channel model of ITS, depending on adversarial noise assumptions, to visualize the trust relations in ZTA workflows, expressing secure channels of communication between pairs of authenticated entities as morphisms while functors would map cryptographic primitives under policy-driven access controls.

3 Algorithm design

A real number $x \in]0, 1]$ can be represented as

$$x = \frac{1}{a_1} + \frac{1}{a_1 a_2} + \frac{1}{a_1 a_2 a_3} + \cdots$$

such that $\{a_1, a_2, \dots\}$ are integers obtained given a seed $x \in]0, 1]$ by the following recursion [2]:

1. The initial point is
$$x_1 = x$$
2. For every $i \geq 1$:
$$a_i = \left\lceil \frac{1}{x_i} \right\rceil,$$
$$x_{i+1} = a_i x_i - 1.$$
When $x_{i+1} = 0$ the algorithm is stopped. Otherwise, we resume once again for $i + 1$.
3. The indices are handed using the logistic map (chaotic [3] to add randomness)
$$j_k = \lfloor N \cdot x_k \rfloor, \quad x_{k+1} = 4x_k(1 - x_k).$$
To shuffle, $a_i \leftrightarrow a_{j_k}$ are swapped for $i = 1, \ldots, N$.

For the Learning With Errors (LWE) encryption [5] let us consider $x \in]0, 1[$ a secret irrational number, p, q are moduli with $q \gg p$, where n the dimension set prior for the lattice, and $\{a_1, a_2, \ldots\}$ the Engel sequence [1].
First, key generation [6] occurs as follows:
$$\mathbf{A} \in \mathbb{Z}_q^{n \times n}, \quad \mathbf{s} \in \mathbb{Z}_q^n, \quad \mathbf{e} \in \mathbb{Z}_q^n.$$

- Let $\mathbf{A}$ be a matrix that is made public
$$\mathbf{A}_{ij} = a_k \mod p \quad \text{for } k \in [\![1, n^2]\!]$$
with $a_k \leq \alpha$ (where α a restricting chosen term) and $\mathbf{s}$ a private key
$$s_i = a_k \mod q \quad \text{for } k \in [\![n^2 + 1, \ldots, n^2 + n]\!]$$
and $\mathbf{e}$ an error vector
$$e_i = \frac{1}{n} \sum_{j=1, j \neq i}^{n} (a_{k_j} - a_{k_i}) \quad \text{for } k \in [\![n^2 + n + 1, \ldots, n^2 + 2n]\!]$$

To encode, let $\overline{e_i} \in \{0, 1\}$ be the mean of $\{e_i\}_{i \in [\![1,n]\!]}$

- Compute a random vector
$$r[i] = a_k \mod 2 \quad \text{for } k \in [\![n^2 + 2n + 1, \ldots, n^2 + 3n]\!]$$
and encrypted text c_1 and c_2
$$c_1 = \mathbf{A}^T \mathbf{r} \mod q, \quad c_2 = \mathbf{b}^T \mathbf{r} + \overline{e_i} \left\lfloor \frac{q}{2} \right\rfloor \mod q$$

The decryption goes as follows
$$\overline{e_i} = \begin{cases} 0 & \text{if } |c_2 - \mathbf{s}^T c_1| < \frac{q}{4} \\ 1 & \text{otherwise} \end{cases}$$

4 Categorical visualization

Let us consider the category D with the objects (X, f) having a set X, an endomorphism $f : X \to X$ and the commutative squares that preserve the dynamics are the morphisms.
The integer sequence $\{a_i\}_{i\in\mathbb{N}}$ forms the objects for the S category and the shuffle transformation on those sequences are the morphisms. The forgetful functor that denotes the Engel expansion is defined as $g : \mathbf{D} \to \mathbf{S}$

$$g\big((]0,1], \phi)\big) = \left\{ \left\lceil \frac{1}{x_i} \right\rceil \right\}_{i\in\mathbb{N}}$$

and the endofunctor $h : \mathbf{S} \to \mathbf{S}$ denotes the shuffle

$$h(\{a_i\}) = \{a_{\sigma(i)}\} \quad \text{with} \quad \sigma \text{ the logistic map-based permutation.}$$

Let $\eta : h \circ g \Rightarrow g \circ h$ ($\circ$ denotes composition) be a natural transformation [8] such that the diagram

$$\begin{array}{ccc} (]0,1], \phi) & \xrightarrow{\quad g \quad} & (\mathbb{N}^{\mathbb{N}}, \psi) \\ \Big\downarrow{\scriptstyle h} & & \Big\downarrow{\scriptstyle h} \\ (]0,1], \phi) & \xrightarrow[\quad g \quad]{} & (\mathbb{N}^{\mathbb{N}}, \psi) \end{array}$$

commutes up to an isomorphism [11].
The fiber product object $(X, \phi) \times_{(\mathbb{N}^{\mathbb{N}},\psi)} (X, \phi)$ represents initial condition x of which the Engel sequences when shuffled they agree under $g \circ h$ and $h \circ g$ with a pullback diagram [9]

$$\begin{array}{ccc} (X, \phi) & \xrightarrow{\quad F \quad} & (\mathbb{N}^{\mathbb{N}}, \psi) \\ \Big\downarrow{\scriptstyle S} & \ulcorner & \Big\downarrow{\scriptstyle S} \\ (X, \phi) & \xrightarrow[\quad F \quad]{} & (\mathbb{N}^{\mathbb{N}}, \psi) \end{array}$$

The fiber co-product $(\mathbb{N}^{\mathbb{N}}, \psi) \sqcup_{(X,\phi)} (\mathbb{N}^{\mathbb{N}}, \psi)$ stands for the Engel sequences with an h shuffle where the pushout diagram [10] is as follows

$$\begin{array}{ccc} (X, \phi) & \xrightarrow{\quad F \quad} & (\mathbb{N}^{\mathbb{N}}, \psi) \\ \Big\downarrow{\scriptstyle F} & \lrcorner & \Big\downarrow{\scriptstyle S} \\ (\mathbb{N}^{\mathbb{N}}, \psi) & \xrightarrow[\quad S \quad]{} & (\mathbb{N}^{\mathbb{N}}, \psi) \end{array}$$

Let $E :]0,1] \to \mathbb{N}^{\mathbb{N}}$ denote the Engel expansion and $C : \mathbb{N}^{\mathbb{N}} \to]0,1]$ denote the convergent series, they both allow a duality adjunction

$$\mathrm{Hom}_{\mathbf{D}}\big((]0,1], \phi), (X, f)\big) \cong \mathrm{Hom}_{\mathbf{S}}\big(\{a_i\}, g(X, f)\big)$$

such that $C \circ E \cong \mathrm{id}_{]0,1]}$ and $E \circ C \cong \mathrm{id}_{\mathbb{N}^{\mathbb{N}}}$.
Considering the monomorphism formed by the one-to-one inclusion $i :]0,1] \hookrightarrow \mathbb{R}$ and the epimorphism that surjects E onto a correct Engel sequence.
The recursion from the expansion series gives a Schönfinkelization currying the function

$$\lambda x.\Big(a(x), \lambda x'.\big(a(x'), \dots\big)\Big) :]0,1] \to \mathbb{N} \times \big(]0,1] \to \mathbb{N} \times \cdots\big)$$

mapping a seed x to a the development function and matching the fixed point $T(\{a_i\}) \cong \{a_i\}$ of an endofunctor $T : \mathbf{D} \to \mathbf{D}$

$$T(X, f) = \left(X \times \mathbb{N}, (x, a) \mapsto \left(f(x), \left\lceil \frac{1}{x} \right\rceil\right)\right)$$

with $x_0 = x \in]0,1]$ as an intial object and

$$\phi(x_i) = (a_i, x_{i+1}) \quad \text{where } a_i = \left\lceil \frac{1}{x_i} \right\rceil, \quad x_{i+1} = a_i x_i - 1$$

as the morphism leading to the D category terminal object $x_{i+1} = 0$.

Let $q > 0$ and $n \in \mathbb{N}$ be the dimension, we consider over $\mathbb{Z}_q$ a structured set of n-dimensional vectors denoted as polyset

$$\mathcal{P}_n \coloneqq q^n = \big\{(x_1, \dots, x_n) \mid x_i \in \{0, \dots, q-1\}\big\}$$

which can represent encrypted text, public matrix or the secret key, with $\mathcal{P}_1 = q$ the message space $\{0,1\}$ scaled by $\lfloor \frac{q}{2} \rfloor$ and $\mathcal{P}_0 = *$ the terminal object.
The probabilistic polynomial-time (PPT) algorithm consisting on the morphism $f : \mathcal{P}_n \to \mathcal{P}_m$ is described as

$$f(\mathbf{x}) = \mathrm{Alg}(\mathbf{x}; \mathbf{r}), \quad \mathbf{r} \leftarrow \mathrm{Sample}(\mathcal{R})$$

where the cryptosystem is

$$\begin{aligned}
&\mathrm{KeyGen} : \mathcal{P}_1 \to \mathcal{P}_n \times \mathcal{P}_n \quad (x \mapsto (\mathbf{A}, \mathbf{s})) \\
&\mathrm{Encrypt} : \mathcal{P}_n \times \mathcal{P}_1 \to \mathcal{P}_n \times \mathcal{P}_1 \quad ((\mathbf{A}, \overline{e_i}) \mapsto (c_1, c_2)) \\
&\mathrm{Decrypt} : \mathcal{P}_n \times \mathcal{P}_n \to \mathcal{P}_1 \quad ((\mathbf{s}, (c_1, c_2)) \mapsto \overline{e_i})
\end{aligned}$$

with the public matrix

$$\mathbf{A}_{ij} = a_{[\![1, n^2]\!]} \mod p \in \mathcal{P}_n$$

and secret key

$$s_i = a_{[\![n^2+1, n^2+n]\!]} \mod q \in \mathcal{P}_n$$

Hence, the encryption becomes a bi-categorical pushout

$$\begin{array}{ccc} \mathcal{P}_n & \xrightarrow{\mathbf{A}} & \mathcal{P}_n \otimes \mathcal{P}_n \\ {\scriptstyle \mathbf{r}}\downarrow & \lrcorner & \downarrow \\ \mathcal{P}_1 & \xrightarrow[\overline{e_i}]{} & \mathcal{C} \end{array}$$

The Yoneda completion [7] embeds the error vector into a functor category:

$$\mathcal{Y}(\mathbf{e})\colon \mathcal{E} \hookrightarrow \mathbf{Set}^{\mathcal{E}^{\mathrm{op}}}$$

this fully faithful embedding yields

$$\begin{array}{ccc} \mathcal{E} & \xrightarrow{\mathcal{Y}} & \mathbf{Set}^{\mathcal{E}^{\mathrm{op}}} \\ {\scriptstyle \mathbf{e}}\downarrow & & \downarrow{\scriptstyle \exists!\mathrm{Decrypt}} \\ \mathbf{Set} & \dashrightarrow & q^n \end{array}$$

with E here denoting the category of Engel expansions and $\mathbf{e}$ becomes universal while leakage attacks should be carried on presheafs. The error vector through Yoneda is

$$e_i = \frac{1}{n}\mathrm{Hom}\left(-, \sum_{\substack{j=1 \\ j\neq i}}^{n} (a_{k_j} - a_{k_i})\right)$$

and to compute the cipher

$$c_1 = \mathbf{A}^T\mathbf{r} \mod q, \quad c_2 = \mathbf{b}^T\mathbf{r} + \overline{e_i}\left\lfloor \frac{q}{2} \right\rfloor \mod q$$

Let $\mathbf{A} \in \mathbf{RPCirc}$ be a public matrix in the category of randomized circuits for probabilistic operations and $\mathbf{s} \in \mathbf{RPCirc}$ a secret key vector with a braiding β from chaotic the logistic map [4], that encodes the shuffling

$$\beta_{\mathrm{enc}}(\mathbf{A} \otimes \mathbf{s}) = \mathbf{s} \otimes h(\mathbf{A})$$

and encrypts using

$$c_1 = \langle \beta_{\mathrm{enc}}(\mathbf{A} \otimes \mathbf{s}), \mathbf{r} \rangle, \quad c_2 = \langle \beta_{\mathrm{enc}}(\mathbf{b}), \mathbf{r} \rangle + \overline{e_i}\lfloor q/2 \rfloor$$

which proves the possibility of decryption since its a commutative braiding diagram (inversibility).

$$\begin{array}{ccc} \mathbf{A} \otimes \mathbf{s} & \xrightarrow{\beta_{\mathrm{enc}}} & \mathbf{s} \otimes \mathbf{A} \\ \cong\downarrow & & \downarrow\cong \\ \mathrm{h}(\mathbf{A}) \otimes \mathbf{s} & \xrightarrow[\beta_{\mathrm{dec}}^{-1}]{} & \mathbf{s} \otimes \mathrm{h}(\mathbf{A}) \end{array}$$

By extension, a dynamical system approximating the behavior of shuffled sequences might be constructed. To define such an extension as a functor $\mathcal{D} \to \mathcal{E}$ with a universal natural transformation

$$\epsilon : \mathrm{Ran}_F K \circ F \Rightarrow K$$

that is called the *right Kan extension* $\mathrm{Ran}_F K$ of K along F , let us consider the two functors $F : \mathcal{C} \to \mathcal{D}$ and $K : \mathcal{C} \to \mathcal{E}$.
Let $\overline{h} : \mathbf{S} \to \mathbf{D}$ be a functor such that

$$\overline{h} \cong \mathrm{Ran}_g(h \circ g)$$

For the constraint $g \circ \overline{h} \cong h \circ g$, the constructed extension is the *best approximate lift* of $h(g(X))$ back in $\mathbf{D}$.

Let us define a presheaf

$$\mathcal{F} \in \widehat{\mathcal{E}}, \quad \mathcal{F}(x) = \big\{(\mathbf{A}, \mathbf{s}, \mathbf{e})/x \in (0,1)\big\}$$

with

$$\mathbf{A}_{ij} = a_k \mod p, \quad s_i = a_{k'} \mod q, \quad e_i = \frac{1}{n}\sum_{j \neq i}(a_{k_j} - a_{k_i})$$

Let Mod be a category of modules over a ring R, the slice category Mod/p is the category with objects $(A, f : A \to p)$ that are modules with maps to a fixed p, and morphisms $A \xrightarrow{g} B$ where $f' \circ g = f$ and consider in this case the objects $(q, \phi : \mathbb{Z}_q \to \mathbb{Z}_p)$ and the projection $\pi : \mathcal{F} \to \mathbf{Mod}$ maps $(q, (\mathbf{A}_q, \mathbf{s}_q, \mathbf{e}_q)) \mapsto q$ such that

$$\begin{array}{ccc} \mathcal{F} & \xrightarrow{\pi} & \mathbf{Mod} \\ {\scriptstyle U}\downarrow & & \downarrow{\scriptstyle I} \\ \mathbf{RPCirc} & \xrightarrow[\mathrm{Ran}_I]{} & \mathbf{Mod}^q \end{array}$$

and the following triangle morphisms are commutative

$$\begin{array}{ccc} (q, \phi_q : \mathbb{Z}_q \to \mathbb{Z}_p) & \xrightarrow{\quad f \quad} & (q', \phi_{q'} : \mathbb{Z}_{q'} \to \mathbb{Z}_p) \\ & {\scriptstyle \phi_q} \searrow \quad \swarrow {\scriptstyle \phi_{q'}} & \\ & \mathbb{Z}_p & \end{array}$$

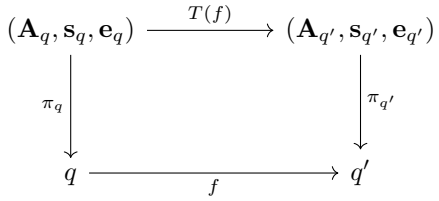
Consider in $\mathbf{Mod}/p$ the morphisms $f : q \to q'$ offering compatible modulus reduction

$$\phi_q = \phi_{q'} \circ f \quad \text{(compatibility of modulus reductions)}.$$

as a consequence $f : \mathbb{Z}_{q'} \to \mathbb{Z}_q$ is reduction modulo q when $q' = kq$ and

$$\phi_q(x \mod q') = \phi_{q'}(x) \mod p.$$

which allows to have a diagram



with $\pi_q(\mathbf{A}_q, \mathbf{s}_q, \mathbf{e}_q) = q$ and

$$\begin{aligned} T(f)(\mathbf{A}_q) &= \mathbf{A}_q \otimes_{\mathbb{Z}_q} \mathbb{Z}_{q'} \\ T(f)(\mathbf{s}_q) &= \mathbf{s}_q \mod q' \\ T(f)(\mathbf{e}_q) &= \mathbf{e}_q \mod q' \end{aligned}$$

Given the universal property emerging from the Kan extension, in $\mathbf{RPCirc}^q$ a unique lifted instance exists such that

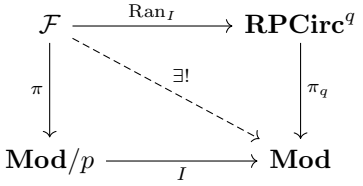
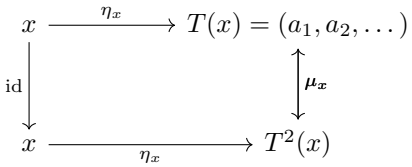


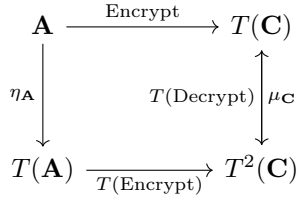
diagram commutes for any LWE instance over q when it is modulus reduction $q \to p$, meaning

$$\forall(\mathbf{A}_p, \mathbf{s}_p) \in \mathcal{F},\ \exists!(\mathbf{A}_q, \mathbf{s}_q) \text{ with } \mathbf{A}_q \equiv \mathbf{A}_p \mod p.$$

The category formed by Engel expansions η_x allows us to view a monad structure



The encryption can be seen as the following Kleisli diagram



To encrypt A let us apply $E_x = \beta(\mathbf{A} \otimes \mathbf{s}) \otimes \mathbf{r}$ with

$$\beta : \mathbf{A}_{ij} \otimes s_j \mapsto s_j \otimes \sigma(\mathbf{A}_{ij})$$

where $E_x(D_x)$ is the encryption fo user x (resp. decryption) and to decrypt $D_x = \beta^{-1}(c \otimes \mathbf{s}) \mod q$ is applied making the Kleisli diagram commutes which means

$$\mu_{\mathbf{C}} \circ T(D_x) \circ T(E_x) = D_x \circ E_x \circ \eta_{\mathbf{A}}$$

5 Zero-Trust Architecture

The traditional setting of network security, represented by VPN, assumes a default trust environment. A user gaining entry into the network, is considered reliable till the end, which is a peculiar shortcoming since this permission grants the attacker lateral movement across the network. The Zero Trust Architecture (ZTA) eliminates the element of trust, requiring constant verification of identity. In this design, post-quantum encryption along with rigid access controls are tailored for safeguarding AI models within a demilitarized zone (DMZ). An external client creates an exclusive cryptographic fingerprint derivable from a secret seed. This is structured using deterministic chaos, specifically a series of integers constructed from a fractional value to provide resistance to quantum attacks. The request from the client gets to be encrypted using the shuffled sequence and identity is embedded in the cryptographic payload. The encrypted request is routed through an ESP32 broker acting as a gatekeeper that decrypts the payload with a pre-shared secret key while also validating the cryptographic proof of the client. Any client device as seen on Figure 1 failing to authenticate is redirected to the public cloud series, thereby completely isolating any access to the private network.

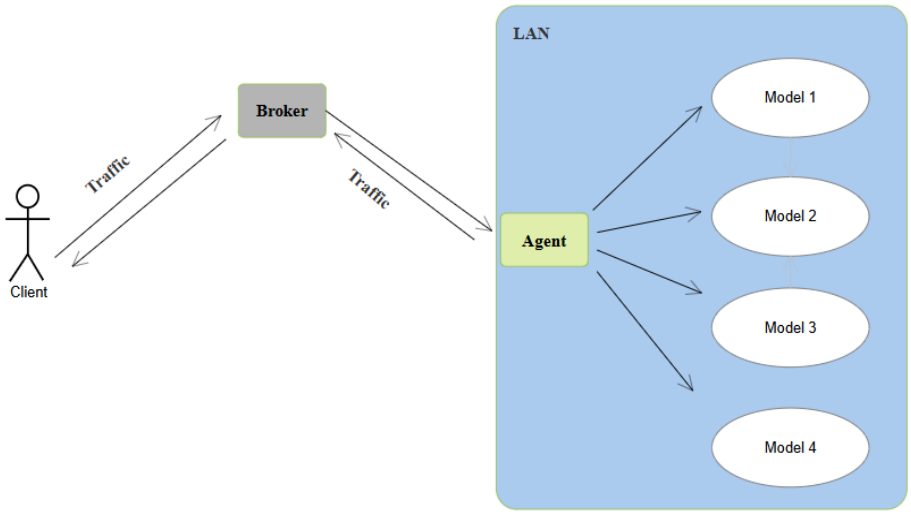


Figure 1: Zero Trust Architecture design for signed using quantum-resistant SSL encryption and time-stamped requests, with a privilege LAN zone separated with access only upon authentication for pre-approved resources.

The approved request goes towards a Zero Trust Agent in the DMZ buffer zone. The Agent functions as a checkpoint validating not only credentials by the client, but also permission contextualization (role, time of day, chosen AI model). Beyond the DMZ, AI models are placed in a LAN zone under lock

down. Access to some models is only based on specific roles of the client and permissions are ephemeral.

6 Results and discussion

The ZTA implementation used ESP32 boards as brokers (authentication: 25 ms) and agents (LWE decryption: 135 ms), with a MacBook acting both as client (LWE encryption: 120 ms) and to host AI services with Hugging Face models: DistilGPT2 (gpt), DistilBERT-base-uncased-finetuned-SST-2-English (bert), and GPT2 (shared by llama and mistral tokens). Token-based micro-segmentation enforced least-privilege access, completely rejecting unauthorized requests (e.g., llama tokens blocked from bert resources) while LWE cryptography (768-dimensional lattice) consumed 12 KB of ESP32 RAM, limiting concurrent clients to 5–8 due to memory constraints and further increasing the agent power draw by 38% (80mA idle → 110mA active). End-to-end latency averaged 760 ms, including AI inference (DistilGPT2: 480 ms, DistilBERT: 12.5 requests/second), and LWE ciphertexts inflated payloads by a factor of 2.1. Security tests verified resistance to chosen-plaintext attacks throughout 1,000 sessions, though model homogeneity (GPT2 reused for llama/mistral) posed some misconfiguration risks. Power traces linked CPU spikes to lattice operations, further confirming computation intensity.

The execution time and cycle count measurements (Table 1 and 2) serve to

Table 1: Agent Execution Time

Task	Avg (s)	Min (s)	Max (s)	Cycles (s)	% Diff
Encryption	10,969.37	9.00	30,156.00	10,972.59	0.03
Decryption	2,890.22	1,653.00	3,500.00	2,891.28	0.04

highlight the cryptographic operations computational efficiency on the ESP32 platform. All cycle-to-time conversions consider the 240 MHz clock speed of ESP32. The difference (0.03 to 0.04%) is negligible between measured time and time derived from cycle counts, hence confirming the accuracy of our time measurement. Minor deviations can be explained by some microsecond delay between cycle count recording and timer capturing. Encryption needs about 3.8 more time (average $10,969 \mu s$) compared to decryption (average $2,890 \mu s$). Large variability between encryption times ($9 \mu s$ to $30 \mu s$) could point to workload fluctuations or perhaps interference from interrupts. The tight correlation between cycles to wall-clock time with error less than 0.05% testifies to the deterministic execution of the ESP32 at 240 MHz, which allows for precise performance benchmarking of real-time applications.

Memory Efficiency for the agent as shown in Table 3 is 91.86% of free heap was remaining after cryptography and after making a request to the AI service. Memory use analysis portrayed great differences among processes. While

Table 2: CPU Cycle Count

Task	Avg (cycles)	Min (cycles)	Max (cycles)
Encryption	2,633,532	3,159	7,239,597
Decryption	694,028	397,876	840,098

decryption presents a very small memory fluctuation (avg. diff 0.067 kB), encryption and request handling use memory much more heavily, especially for requests (avg. diff of 18.96 kB and max of 43.26 kB).

The broker's memory efficiency (Table 4), with 97.88% of the free heap remaining, is better than that of the agent, suggesting that handling responses is memory-optimized for the broker. Quite notably, the request process exhibits the most memory fluctuation, possibly indicating its dynamic allocation patterns during AI service interactions. While encryption exhibits moderate memory reduction (average of 5.72 kB decrease), this could have been due to temporary buffer allocations during the cryptographic operations. The average response time from the agent is 3085.89 ms and when the broker is communicating with the agent it includes network latency from requests and time taken for the AI service to process the prompts.

Table 3: Agent Memory Usage

Process	Avg Before (kB)	Avg After (kB)	Avg Diff (kB)	Max Diff (kB)
Decryption	202.25	202.18	0.067	0.084
Encryption	200.71	195.041	5.72	15.78
Request	203.16	185.78	18.96	43.26

Table 4: Broker Memory Usage

Process	Avg Before (kB)	Avg After (kB)	Avg Diff (kB)	Max Diff (kB)
Response	222.3635	217.65	4.71	7.94

Power traces (Table 5) capture significant fluctuations during these encryption frames, with observed peak power consumption surpassing average levels by 33.92% and 56.55%. The measurements were taken in a very well-instrumented setup with differential probe traces voltage across a 1Ω resistor placed in series with the ESP32's 5 V supply so that the current can be derived using Ohm's Law. The encryption procedure is triggered by toggling a GPIO and the oscilloscope captures are made synchronous with the voltage edge trigger 1.7 V). The spikes discerned in power consumption correspond to heavy cryptographic operations for encrypting AI service responses, which makes a significant increase in the board's current draw. Thus, the 5 V input contributes to the peculiar power profile while the chip operates at 3.3 V, thanks to energy dissipation across the

Table 5: Oscilloscope Power Traces During Encryption

Trace	Avg Power (mW)	Peak Power (mW)	Power Diff (mW)	% Difference
1	302.5	405.1	102.6	33.92
2	302.5	469.3	166.8	55.14
3	304.5	476.7	172.2	56.55

voltage regulator. The fairly uniform power averaged across the traces (302.5-304.5 mW) reflects the steady state baseline, while the oscillating peak power (405.1 - 476.7 mW) reveals the changes associated with the energetic cryptographic computations.

The power trace on Figure 2 for the model llama depicts the existence of a 60.0 mA baseline consumption at 300 mW, with an average consumption out to 60.7 mA or 303.4 mW and a max measured consumption value of 95.8 mA or 479.1 mW. These peaks indicate a 33.92% increase from the average, a hallmark for intermittent burst computation. Important encryption operations take place at about 3 ms, 5.5 ms, and 8 ms, with the spike at 8 ms being highest tallies with heavy computational activities like lattice-based arithmetic-laced LWE encryption. Small baseline fluctuations, such as one at 2 ms, probably correspond to some side work like memory management. The scattered spikes point toward batch processing of cryptographic operations, thereby distinguishing llama from the other models.

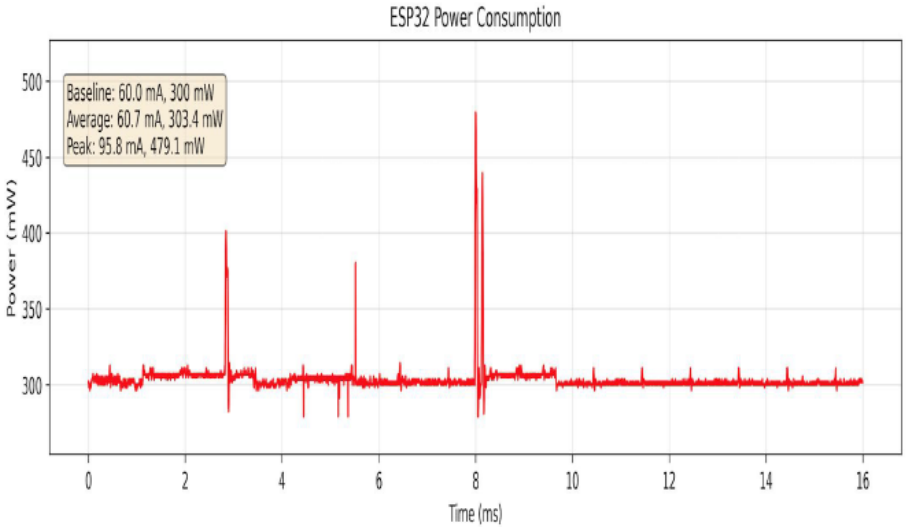


Figure 2: **Power Consumption Trace for Llama Model Encryption**: oscilloscope capture (10 mV/div, 1 ms/div) of ESP32 power draw during the LWE encryption of Llama model responses. Baseline: 60.0 mA (300 mW), Average: 60.7 mA (303.4 mW), Peak: 95.8 mA (479.1 mW) (33.9% above average). Distinct computational phases at 3 ms (key initialization), 5.5 ms (intermediate arithmetic), and 8 ms (intensive lattice operations) are clearly visible. Baseline fluctuations (i.e., around 2 ms dip) signify background memory management. The spaced spike intervals hint at batched cryptographic processing.

The gpt model trace on Figure 3 has a baseline of 60.0 mA (300 mW), an average of 60.5 mA (300.5 mW), and a peak of 93.9 mA (469.3 mW), with a 55.14% peak-to-average difference-the highest among traces. Encryption events are clustered around the 5.5 ms, 6.8 ms, and 11 ms regions, with the 11 ms peak being the highest. The closeness of these peaks indicates rapid-fire sequential operations, possibly additional rounds of encryption for longer messages. The 6.8 ms delay might have been due to interrupt processing or cache misses related to memory management. This trace is a representation of pipelined computation as opposed to the batched processing of llama.

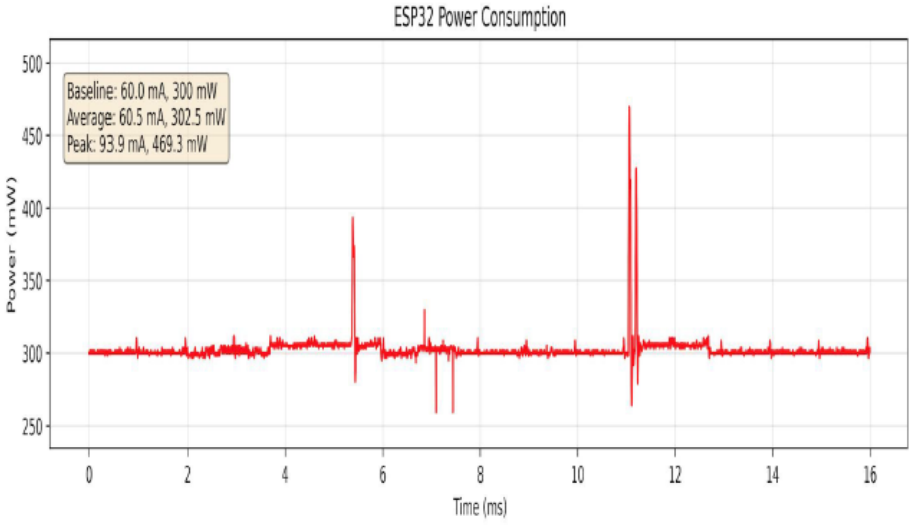


Figure 3: **Power Consumption Trace for GPT Model Encryption**: the power profile (10 mV/div, 1 ms/div) of ESP32 during the encryption of GPT model responses. Baseline: 60.0 mA (300 mW), Average: 60.5 mA (300.5 mW), Peak: 93.9 mA (469.3 mW) (55.1% above average). The cluster of spikes at 5.5 ms (parallelizable ops), 6.8 ms (potential cache miss), and 11 ms (multi-round encryption) mark pipelined LWE computations. A higher peak-to-average ratio as compared to Llama suggests that more encryption rounds are being carried out for longer messages. Variations in timing suggest dynamic scheduling on the load.

In the mistral setup, the baseline power draw is 60.0 mA (300 mW), while on average, it draws 60.9 mA (304.5 mW), and at spikes, it goes up to 95.3 mA (476.7 mW), with a peak-to-average difference of 56.55%. The trace on Figure 4 resembles one of gpt, with spikes at 5.8 ms and 11 ms: this is really strong evidence that the messages might both have very similar lengths or that the computational paths are alike. The 11 ms spike marks an episode of heavy computation; meanwhile, the slight shift in timings, 5.8 ms as opposed to gpt's 5.5 ms, might be attributed to pipeline stalls. The resemblance of Traces 2 and 3 corroborates their similarity in workload, likely due to a common model architecture or encryption demand.

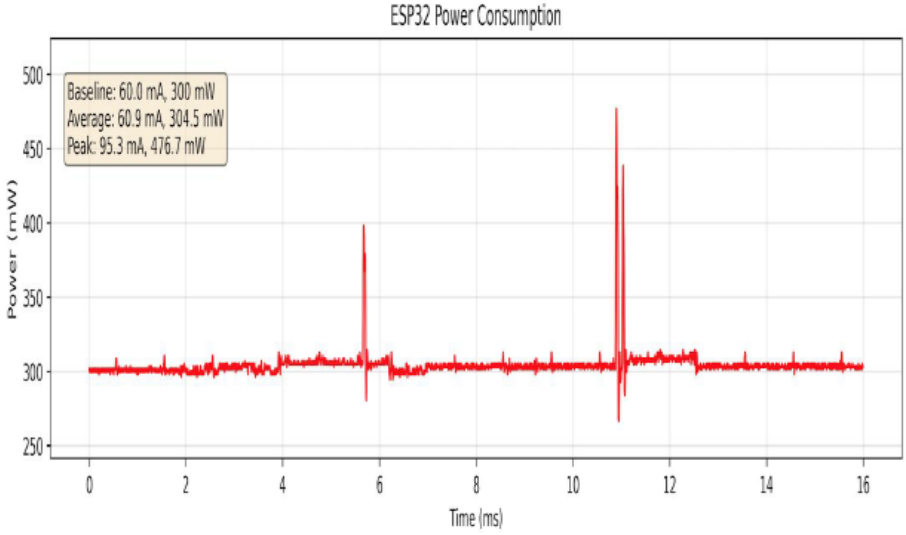


Figure 4: **Power consumption trace for mistral model cryptography operation**: the power trace from ESP32 (10 mV/div, 1 ms/div) during Mistral model encryption showed mice behavior. Baseline: 60.0 mA (300 mW), Average: 60.9 mA (304.5 mW), Peak: 95.3 mA (476.7 mW) (56.6% above average). Key events at 5.8 ms (slightly delayed vs GPT because of pipeline stalls) and 11 ms (dominant lattice ops). Peak timing and magnitude were near-identical to GPT (1.1% difference), strongly asserting that the computational patterns were shared, probably owing to shared message structures or model architectures.

The traces allow us to infer some model-specific encryption characteristics: llama tends toward atypical batched operations, whereas gpt and mistral exhibit pipelined workloads with tighter spike clustering. Peak power differences scale with message length and computational intensity and thus suggest means of optimizing LWE cryptography on resource-constrained hardware like the ESP32.

7 Conclusion

The study builds and exemplifies a ZTA with the support of token-based authentication, IP whitelisting, and LWE encryption enforcement matrices on ESP32s to strictly ensure access control and confidentiality of AI models. Experimental results indicate the practical use of lattice cryptography on low-end systems where encryption takes an average of 10.97 ms, decryption of 2.89 ms, and a bare minimum effect on overall latency, where dominant latencies arise from network delays and AI inference. Power analysis highlighted a baseline constant power of 300 mW with peaks up to 479 mW, fluctuating according to message complexity, peak-to-average variations (33.92%-56.55%) agreeing with cryptographic workloads expectations. Network resilience was kept intact with auto-

reconnect coupled with retry and request cache; meanwhile, client-specific LWE keys secured the communication path end-to-end. Demonstrating an ESP32 enforcing ZTA-minimal overhead fine up to 3ms added latency across llama, gpt, and mistral, this layout is the proof of LWE in practice with power traces that defused computational load. Potential developments entail enhanced transport security with SSL/TLS, advanced error handling, and dynamic key rotation to harden it against threats. Overall, this provides a quantum-resistant and scalable lifetime for our AI system.

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