PKE and ABE with Collusion-Resistant Secure Key Leasing

Fuyuki Kitagawa[†♢], Ryo Nishimaki[†♢], Nikhil Pappu[*⋆]

[†]NTT Social Informatics Laboratories, Tokyo, Japan
{fuyuki.kitagawa,ryo.nishimaki}@ntt.com
[♢]NTT Research Center for Theoretical Quantum Information, Atsugi, Japan
[⋆]Portland State University, USA
nikpappu@pdx.edu

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Abstract

Secure key leasing (SKL) is an advanced encryption functionality that allows a secret key holder to generate a quantum decryption key and securely lease it to a user. Once the user returns the quantum decryption key (or provides a classical certificate confirming its deletion), they lose their decryption capability. Previous works on public key encryption with SKL (PKE-SKL) have only considered the single-key security model, where the adversary receives at most one quantum decryption key. However, this model does not accurately reflect real-world applications of PKE-SKL. To address this limitation, we introduce *collusion-resistant security* for PKE-SKL (denoted as PKE-CR-SKL). In this model, the adversary can adaptively obtain multiple quantum decryption keys and access a verification oracle which validates the correctness of queried quantum decryption keys. Importantly, the size of the public key and ciphertexts must remain independent of the total number of generated quantum decryption keys. We present the following constructions:

- A PKE-CR-SKL scheme based on the learning with errors (LWE) assumption.
- An attribute-based encryption scheme with collusion-resistant SKL (ABE-CR-SKL), also based on the LWE assumption.
- An ABE-CR-SKL scheme with classical certificates, relying on multi-input ABE with polynomial arity.
- An adaptively secure identity-based encryption scheme with collusion-resistant SKL based on the LWE assumption.

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1 Introduction

Secure key leasing. Encryption with secure key leasing (SKL) enables a secret key holder to generate a quantum decryption key and lease it securely to another party. Once the lessee returns the quantum decryption key, they lose their ability to decrypt ciphertexts. Since its introduction by Kitagawa and Nishimaki for secret key functional encryption (SKFE) [KN22], SKL has been extensively studied [AKN+23, APV23, CGJL23, MPY23, AHH24, BGK+24, KMY24] due to its strong security guarantee and practical applications.

Collusion-resistant SKL. Most prior works [AKN+23, APV23, CGJL23, MPY23, AHH24] study how to achieve public-key encryption (PKE) with SKL schemes from standard cryptographic assumptions. All these prior works on PKE-SKL have explored the setting where an adversary can obtain *only one* quantum decryption key. However, this single-key security model does not accurately reflect real-world scenarios. In practice, once a lessee returns their decryption key and it is verified (i.e., after revocation), the lessor may lease another decryption key, even to the same lessee. Moreover, in realistic settings, a single secret key holder may need to generate and lease *multiple* quantum decryption keys to various entities. To accurately capture this setting, we consider adversaries capable of obtaining *an unbounded number of quantum decryption keys*, even in the standard PKE setting. We define this model as *collusion-resistant* SKL.

Previous works [APV23, AHH24] presented delegation tasks as a compelling application like the following: Consider a scenario where a system administrator unexpectedly needs to take leave and must temporarily assign their responsibilities—including access to encrypted data—to a colleague by providing decryption keys. In such cases, the single-key security model is inadequate, as the administrator may need to take leave multiple times or assign their responsibilities to different colleagues. Another potential application of collusion-resistant PKE-SKL is in streaming services. Encrypted videos are made accessible to subscribers through quantum decryption keys. When a user unsubscribes, they return their leased keys, losing access to the content, and their subscription fees are canceled. By utilizing an attribute-based encryption [SW05] variant of collusion-resistant PKE-SKL, it becomes possible to precisely control video access based on user attributes, such as location-restrictions or premium and basic subscription plans.

Our goal: collusion-resistant SKL from weaker assumptions. The notion of collusion-resistant SKL is both natural and compelling. However, it has yet to be achieved from well-established assumptions. All known PKE with SKL schemes based on standard assumptions [AKN+23, APV23, CGJL23, AHH24, KMY24] do not remain secure in the collusion-resistant setting. While some existing constructions seem to imply collusion-resistant SKL, such as public-key functional encryption (PKFE) schemes with SKL [AKN+23, BGK+24] and collusion-resistant single decryptor encryption (SDE)[1] [ÇG24], they rely on strong assumptions. These include post-quantum secure indistinguishability obfuscation (IO) or collusion-resistant PKFE, which in turn implies IO, albeit with a sub-exponential security loss [BV18, AJ15, AJS15]. Achieving them from well-established assumptions still remains elusive. In this work, we aim to construct the first collusion-resistant SKL schemes based on weaker assumptions.

1.1 Our Results

Our main contributions are summarized as follows:

1. *Definition of collusion-resistant PKE-SKL (PKE-CR-SKL):* We formally define PKE-CR-SKL, ensuring that if all quantum decryption keys are returned and successfully verified, users lose decryption capabilities. We extend the indistinguishability against key leasing attacks (IND-KLA) security definition [AKN+23] to the collusion-resistant setting. One notable feature of this definition is that the adversary can send multiple queries to the verification oracle, which confirms the validity of returned decryption keys. Another important feature is that the public key and ciphertext size are independent of the number of leased decryption keys (up to logarithmic factors).

[1] In short, SDE is PKE where the decryption keys are copy-protected (i.e., unclonable). In general, SDE implies encryption with SKL (see the discussion by Agrawal et al. [AKN+23] for details).

Table 1: Comparison of SKL. VO means security in the presence of the verification oracle for certificates.

	Primitive	Collusion-resistant SKL	VO	Certificate	Assumption
[AKN+23]	PKE	–	✓	quantum	PKE
[AKN+23]	ABE	bounded	✓	quantum	ABE
[AKN+23]	bCR PKFE[a]	bounded	✓	quantum	PKE
[AKN+23]	PKFE	✓	✓	quantum	PKFE[b]
[BGK+24]	PKFE[c]	✓	✓[d]	classical	IO
[APV23, AHH24]	PKE (FHE)	–	–	classical	LWE
[CGJL23]	PKE (FHE)	–	–	classical	LWE
[KMY24]	PKE	–	✓[e]	classical	PKE
Ours1	PKE	✓	✓	quantum	LWE
Ours2	ABE[c]	✓	✓	quantum	LWE
Ours3	PKE	✓	✓	classical	MI-ABE
Ours4	ABE[c]	✓	✓	classical	MI-ABE
Ours5	IBE[f]	✓	✓	quantum	LWE

a bCR denotes bounded collusion-resistant. This scheme only satisfies the bounded collusion resistance of standard PKFE.
b Collusion-resistant PKFE implies IO up to sub-exponential security loss.
c These schemes are selectively secure, where adversaries must declare the target plaintexts (PKFE case) or attributes (ABE case) at the beginning.
d This scheme has public verifiability (the verification key for certificates is public).
e This scheme is secure even if the verification key is revealed to the adversary *after* the adversary outputs a valid certificate.
f This scheme is adaptively secure, where adversaries declare the target identity when they decide the target plaintexts.

2. *Construction of IND-KLA secure PKE with collusion-resistant SKL:* We propose an IND-KLA secure PKE-CR-SKL scheme based on the learning with errors (LWE) assumption.

3. *Attribute-based encryption with collusion-resistant SKL:* We construct an ABE-CR-SKL scheme, also based on the LWE assumption. Since PKE is a special case of ABE, ABE-CR-SKL trivially implies PKE-CR-SKL. We first present the PKE-CR-SKL scheme separately to provide a clearer foundation for understanding the ABE-CR-SKL construction.

4. *PKE and ABE with collusion-resistant SKL and classical certificates:* We also propose an IND-KLA secure ABE-CR-SKL scheme that utilizes *classical* certificates, whereas the constructions above rely on quantum certificates.[2] In this model, a classical certificate can be derived from a leased quantum decryption key, and successful verification guarantees security. Our scheme is based on multi-input ABE (MI-ABE), which is a potentially weaker assumption than collusion-resistant PKFE. We specify required properties for MI-ABE and discuss the relationship between MI-ABE and other primitives in the full version of this work [KNP25]. ABE-CR-SKL with classical certificates trivially implies PKE-CR-SKL with classical certificates.

5. *Adaptively secure identity-based encryption with collusion-resistant SKL (IBE-CR-SKL):* We construct an adaptively secure IBE-CR-SKL scheme based on the LWE assumption. In the adaptive security model, adversaries select the target identity when they decide target plaintexts. This scheme is not an immediate corollary of the ABE-CR-SKL schemes above since they are *selectively* secure.

We introduce fascinating techniques to achieve our results. These include the classical decryption property and the notion of key-testability for secret key encryption with collusion-resistant SKL (SKE-CR-SKL) and secret key functional encryption with collusion-resistant SKL (SKFE-CR-SKL). Then, we transform SKE-CR-SKL (resp. SKFE-CR-SKL) into PKE-CR-SKL (resp. ABE-CR-SKL) by using classical ABE and compute-and-compare obfuscation [WZ17, GKW17]. In these transformations, we use decryption keys of SKE-CR-SKL (resp. SKFE-CR-SKL) as key attributes of ABE in a superposition way. See Section 2 for the details.

[2] Quantum decryption keys function as quantum certificates.

1.2 Related Work

Encryption with SKL. Agrawal et al. [AKN+23] presented PKE-SKL, ABE-SKL, and PKFE-SKL schemes. Their PKE-SKL scheme is based on standard PKE, and its certificate is quantum. This scheme is not collusion-resistant. Their ABE-SKL scheme is based on PKE-SKL and standard (collusion-resistant) ABE, and its certificates are ones of the underlying PKE-SKL. This scheme is bounded collusion-resistant, that is, the adversary can obtain an a-priori bounded number of quantum decryption keys that can decrypt target ciphertexts. Their PKFE-SKL scheme is based on PKE-SKL and standard (collusion-resistant) PKFE, and its certificates are ones of the underlying PKE-SKL. This scheme is collusion-resistant. (If we instantiate the PKFE-SKL scheme with bounded collusion-resistant PKFE, the scheme is bounded collusion-resistant with respect to both standard PKFE and SKL.) All schemes mentioned above are secure in the presence of the verification oracle for certificates.

Bartusek et al. [BGK+24] also presented a PKFE-SKL scheme. Their scheme is collusion-resistant in the selective model, where the target plaintext is declared at the beginning of the game. In addition, their certificates are classical and publicly verifiable. However, their scheme relies on IO.

Ananth, Poremba, and Vaikuntanathan [APV23] presented a PKE-SKL scheme with classical certificates based on the LWE assumption and an unproven conjecture. Since the encryption algorithm of this scheme is essentially the same as the Dual-Regev PKE scheme [GPV08], their scheme achieves fully homomorphic encryption (FHE) with SKL. Later, Ananth, Hu, and Huang [AHH24] present a new security analysis for Ananth et al.'s scheme and removed the conjecture. Chardouvelis, Goyal, Jain, and Liu [CGJL23] presented a PKE-SKL scheme with classical certificates based on the LWE assumption. Their scheme is based on the Regev PKE scheme [Reg09], so it also achieves FHE with SKL. In addition, their scheme also achieves classical communication, where all messages between senders and receivers are classical. Kitagawa, Morimae, and Yamakawa [KMY24] presented a PKE-SKL scheme with classical certificates based on PKE. These three works do not achieve collusion resistance. Ananth et al. [APV23, AHH24] and Chardouvelis et al. [CGJL23] do not consider the verification oracle in their security definitions, while Kitagawa et al. [KMY24] does.[3] We summarize the works on encryption with SKL in Table 1.

Secure software leasing. Ananth and La Placa [AL21] introduced secure software leasing, which encodes classical programs into quantum programs that we can securely lease. We can view SKL as secure software leasing for decryption functions. However, previous works on secure software leasing consider a sub-class of evasive functions [AL21, CMP20, KNY21, BJL+21] or PRFs [KNY21], which do not support decryption functions. Moreover, they consider a weak security model in which pirated programs use *honest* evaluation algorithms [AL21, KN23, BJL+21] or rely on the quantum random oracle model [CMP20]. Bartusek et al. [BGK+24] achieve secure software leasing supporting all differing inputs circuits[4] in a strong security model where pirated programs can use arbitrary evaluation algorithms. However, their scheme relies on IO.

Multi-input ABE. Roughly speaking, MI-ABE is ABE that can support multiple ciphertext-attributes (or multiple key-attributes). To achieve our classical certificates scheme, we need MI-ABE for polynomial-size circuits where the number of slots is polynomial, and we can generate ciphertexts for one slot using a public key. See our full version [KNP25] for the definition. However, as we review previous works on MI-ABE for general circuits[5] below, none of them achieves what we need without using IO.[6]

Agrawal, Yadav, and Yamada [AYY22] proposed *two-input* ABE for polynomial-size circuits based on lattices. However, the scheme is heuristic (no reduction-based security proof) and needs a master *secret key to generate ciphertexts for all slots*. Agrawal, Rossi, Yadav, and Yamada [ARYY23] proposed MI-ABE for NC^1 from the evasive LWE assumption and MI-ABE for polynomial-size circuits from the evasive LWE and tensor LWE assumptions. Their scheme allows to generate ciphertexts for one slot using a public key. However, the number of slots is *constant*. Agrawal, Kumari, and Yamada [AKY24] proposed MI-ABE[7] for polynomial-size circuits based on the evasive LWE assumption.

[3] More precisely, the work considers adversaries that receive a verification key after they output a valid certificate. Kitagawa et al. show that their scheme can be converted to satisfy the IND-KLA security by Agrawal et al. [AKN+23].

[4] Roughly speaking, a pair of circuits (C_0, C_1) is differing input if it is hard to find an input y such that $C_0(y) \neq C_1(y)$.

[5] Francati, Fior, Malavolta, and Venturi [FFMV23] and Agrawal, Tomida, and Yadav [ATY23] proposed MI-ABE for restricted functionalities.

[6] IO implies multi-input functional encryption [GGG+14], which implies MI-ABE.

[7] Precisely speaking, their scheme is predicate encryption, which satisfies privacy for attributes.

In their scheme, the number of slots is polynomial. However, we need a master *secret key to generate ciphertexts for all input-attributes*.

Broadcast encryption. Broadcast encryption [FN94] enables a sender to generate ciphertexts intended for a specific subset of users. Only the designated users can decrypt the ciphertexts, while even if all other users collude, they cannot recover the message. A key performance metric for broadcast encryption is the size of the public key and ciphertexts. Several works have proposed optimal broadcast encryption schemes, where these sizes are $\mathrm{poly}(\log N)$ and N is the total number of users. The constructions by Agrawal and Yamada [AY20] and Agrawal, Wichs, and Yamada [AWY20] rely on the LWE assumption and *pairings*, which are not post-quantum secure. The construction by Wee [Wee22] relies on the *evasive LWE assumption*, which is a non-falsifiable assumption and has known counterexamples [BÜW24]. The construction by Brakerski and Vaikuntanathan [BV22] relies on *heuristics* and lacks a reduction-based proof.

While broadcast encryption is particularly well-suited for streaming services, PKE-CR-SKL can also be applied in this domain. Each approach has its own advantages and limitations. One advantage of broadcast encryption is that it allows the sender to specify recipients at the encryption phase, whereas PKE-CR-SKL does not. However, PKE-CR-SKL offers three notable advantages over optimal broadcast encryption.

1. **Efficient revocation:** Since decryption keys in broadcast encryption are classical, the system requires maintaining a revocation list, which senders use to revoke users, whereas PKE-CR-SKL eliminates the need for such lists.
2. **Seamless user expansion:** In broadcast encryption, the user set is fixed during the setup phase, meaning that adding new users requires updating the encryption key. In contrast, PKE-CR-SKL allows new users to be added without requiring any key updates.
3. **Weaker cryptographic assumptions:** Our PKE-CR-SKL scheme is based on the standard LWE assumption, whereas post-quantum secure optimal broadcast encryption relies on the evasive LWE assumption, which has counterexamples [BÜW24].

In terms of asymptotic efficiency, both optimal broadcast encryption and PKE-CR-SKL achieve the same public key and ciphertext sizes.

Certified deletion. Broadbent and Islam [BI20] introduced encryption with certified deletion, where we can generate classical certificates to guarantee that *ciphertexts* were deleted. Subsequent works improved Broadbent and Islam's work and achieved advanced encryption with certified deletion [HMNY21, Por23, BK23, HKM$^+$24, BGK$^+$24] and publicly verifiable deletion [HMNY21, BGK$^+$24, KNY23, BKM$^+$23]. Compute-and-compare obfuscation with certified deletion introduced by Hiroka et al. [HKM$^+$24] is essentially the same as secure software leasing in the strong security model.

Single decryptor encryption. Georgiou and Zhandry [GZ20] introduced the notion of SDE. They constructed a public-key SDE scheme from one-shot signatures [AGKZ20] and extractable witness encryption with quantum auxiliary information [GGSW13, GKP$^+$13]. Coladangelo, Liu, Liu, and Zhandry [CLLZ21] constructed a public-key SDE scheme from IO [BGI$^+$12] and extractable witness encryption or from subexponentially secure IO, subexponentially secure OWF, and LWE by combining the results by Culf and Vidick [CV22]. Kitagawa and Nishimaki [KN22] introduced the notion of single-decryptor functional encryption (SDFE), where each functional decryption key is copy-protected and constructed single decryptor PKFE for $\mathsf{P/poly}$ from the subexponential hardness of IO and LWE. These works consider the setting where the adversary receives only one copy-protected decryption key. Liu, Liu, Qian, and Zhandry [LLQZ22] study SDE in the collusion-resistant setting, where the adversary receives multiple copy-protected decryption keys. They constructed a public-key SDE scheme with bounded collusion-resistant copy-protected keys from subexponentially secure IO and subexponentially secure LWE.

Multi-copy revocable encryption. Ananth, Mutreja, and Poremba [AMP24] introduced multi-copy revocable encryption. This notion considers the setting where we can revoke *ciphertexts* (not decryption keys), and the adversary receives multiple copies of the target *ciphertext* (they are pure states). Hence, this notion is different from secure key leasing (or key-revocable cryptography).

2 Technical Overview

We now provide a technical overview of this work. Our primary focus here is on constructing PKE-CR-SKL and ABE-CR-SKL based on the LWE assumption. For an overview of our variants with classical certificates based on MI-ABE and adaptively secure IBE-CR-SKL scheme, please refer to Sections 8.3 and 9.2, respectively.

2.1 Defining PKE with Collusion-Resistant SKL

We will begin by describing the definition of PKE with Secure Key-Leasing (PKE-SKL) [AKN+23, APV23], and then get into our collusion-resistant generalization of it. PKE-SKL is a cryptographic primitive consisting of five algorithms: Setup, $\mathcal{KG}$, Enc, $\mathcal{Dec}$ and $\mathcal{Vrfy}$. The algorithm Setup samples a public encryption-key ek and a "master" secret-key msk. The encryption algorithm Enc takes a message m and ek as inputs and produces a corresponding ciphertext ct. Both these algorithms are classical and similar to their counterparts in standard PKE. On the other hand, the key-generation algorithm $\mathcal{KG}$ is quantum, and produces a pair of keys $(\mathit{dk}, \mathsf{vk})$ given msk as input. Here, dk is a quantum decryption-key using which $\mathcal{Dec}$ can decrypt arbitrarily many ciphertexts encrypted under ek. The other key vk is a classical verification-key, the purpose of which will be clear in a moment.

The setting to consider is one where an adversary is given a decryption-key dk, and is later asked to return it back. Intuitively, we wish to guarantee that if dk is returned, the adversary can no longer decrypt ciphertexts encrypted under ek. Since a malicious adversary may even send a malformed key $\widetilde{\mathit{dk}}$, we need a way to tell whether the decryption-key has been correctly returned. This is the purpose of the algorithm $\mathcal{Vrfy}$, which performs such a check using the corresponding (private) verification-key vk. It is required that if the state dk is sent back undisturbed, then $\mathcal{Vrfy}$ must accept. On the other hand, if $\mathcal{Vrfy}$ accepts (even for a possibly malformed state), then the adversary must lose the ability to decrypt. This loss in the ability to decrypt is captured formally by a cryptographic game, where an adversary is asked to distinguish between ciphertexts of different messages, after passing the verification check.

So far, we have described the notion of PKE-SKL. In this work, we study the notion of PKE with Collusion-Resistant SKL (PKE-CR-SKL). This primitive has the same syntax as PKE-SKL, but the aforementioned security requirement is now stronger. Specifically, an adversary that obtains polynomially-many decryption keys and (verifiably) returns them all, should no longer be able to decrypt. This is defined formally in our notion of IND-KLA (Indistinguishability under Key-Leasing Attacks) security (Definition 4.3). As standard in cryptography, it is characterized by a game between a challenger $\mathcal{Ch}$ and a QPT adversary $\mathcal{A}$. An informal description of the game follows:

IND-KLA Game in the Collusion-Resistant Setting

1. $\mathcal{Ch}$ samples $(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ and sends ek to $\mathcal{A}$.
2. Then, $\mathcal{A}$ requests q decryption keys corresponding to ek, where q is some arbitrary polynomial in λ.[8]
3. $\mathcal{Ch}$ generates $(\mathit{dk}_i, \mathsf{vk}_i)_{i\in[q]}$ using q independent invocations of $\mathcal{KG}(\mathsf{msk})$. It sends $\{\mathit{dk}_i\}_{i\in[q]}$ to $\mathcal{A}$.
4. Corresponding to each index $i \in [q]$, $\mathcal{A}$ is allowed oracle access to the algorithm $\mathcal{Vrfy}(\mathsf{vk}_i, \cdot)$ in the following sense: For a quantum state $\widetilde{\mathit{dk}}$ queried by $\mathcal{A}$, the oracle evaluates $\mathcal{Vrfy}(\mathsf{vk}_i, \widetilde{\mathit{dk}})$ and measures the verification result. It then returns the obtained classical outcome (which indicates accept/reject). We emphasize that $\mathcal{A}$ is allowed to interleave queries corresponding to indices $i \in [q]$, and can also make its queries adaptively.
5. If at-least one query of $\mathcal{A}$ to $\mathcal{Vrfy}(\mathsf{vk}_i, \cdot)$ produces an accept output for every $i \in [q]$, the game proceeds to the challenge phase. Otherwise, the game aborts.
6. In the challenge phase, $\mathcal{A}$ specifies a pair of messages $(\mathsf{m}_0, \mathsf{m}_1)$. $\mathcal{Ch}$ sends $\mathsf{Enc}(\mathsf{ek}, \mathsf{m}_{\mathsf{coin}})$ to $\mathcal{A}$ for a random bit coin.
7. $\mathcal{A}$ outputs coin'.

$\mathcal{A}$ wins the game whenever $\mathsf{coin} = \mathsf{coin}'$. The security requirement is that for every QPT adversary, the winning probability conditioned on the game not aborting is negligibly close to $1/2$. Observe that the adversary is allowed to

[8] Without loss of generality, we can assume that $\mathcal{A}$ asks for sufficiently many keys at the beginning itself. Hence, even adversaries that can request additional keys after accessing the verification oracle are covered by this definition.

make polynomially-many attempts in order to verifiably return a decryption-key $d\mathcal{K}_i$, and unsuccessful attempts are not penalized. As long as the adversary verifiably returns all the q keys (checked in Step 5. above), the game proceeds. We also emphasize that in the definition, q is an unbounded polynomial, i.e., the construction is not allowed to depend on q in any way.

2.2 Insecurity of Direct Extensions of Prior Work

Next, we will provide some intuition regarding the insecurity of direct extensions of prior works to this stronger setting.

Let us consider the PKE-SKL scheme due to Agrawal et al. [AKN+23] for demonstration. The decryption-keys in their scheme are of the following form[9]:

$$\frac{1}{\sqrt{2}}\left(|0\rangle |\mathsf{sk}_0\rangle + |1\rangle |\mathsf{sk}_1\rangle \right)$$

Here, $\mathsf{sk}_0, \mathsf{sk}_1$ are secret-keys corresponding to public-keys $\mathsf{pk}_0, \mathsf{pk}_1$ respectively of a standard PKE scheme. Specifically, the pairs $(\mathsf{pk}_0, \mathsf{sk}_0)$ and $(\mathsf{pk}_1, \mathsf{sk}_1)$ are generated using independent invocations of the setup algorithm of PKE. The PKE-SKL public-key consists of the pair $(\mathsf{pk}_0, \mathsf{pk}_1)$. The encryption algorithm outputs ciphertexts of the form $(\mathsf{ct}_0, \mathsf{ct}_1)$, where for each $i \in \{0,1\}$, ct_i encrypts the plaintext under pk_i. The verification procedure requires the adversary to return the decryption-key, and checks if it is the same as the above state. The intuition is that if the adversary retains the ability to decrypt, then it cannot pass the verification check with probability close to 1. For instance, measuring the state provides the adversary with sk_0 or sk_1 which is sufficient for decryption, but it clearly destroys the above quantum state.

Consider now the scenario where the public-key $(\mathsf{pk}_0, \mathsf{pk}_1)$ is fixed, and $n = \mathrm{poly}(\lambda)$ copies of the above decryption-key are given to an adversary. In this case, it is easy to see that the adversary can simply measure the states to obtain both sk_0 and sk_1. Even though this process is destructive, since sk_0 and sk_1 completely describe the state, the adversary can just recreate many copies of it and pass the deletion checks. Consequently, one might be tempted to encode other secret information in the Hadamard basis by introducing random phases. However, we observe that such approaches also fail due to simple gentle-measurement attacks. For example, consider the combined state of the n decryption-keys in such a case:

$$\frac{1}{2^{n/2}}\Big(|0\ldots0\rangle |\mathsf{sk}_0 \ldots \mathsf{sk}_0\rangle - |00\ldots1\rangle |\mathsf{sk}_0\mathsf{sk}_0 \ldots \mathsf{sk}_1\rangle + \cdots - |1\ldots1\rangle |\mathsf{sk}_1 \ldots \mathsf{sk}_1\rangle \Big)$$

Clearly, only the last term of the superposition does not contain sk_0, and the term only has negligible amplitude. Hence, one can compute sk_0 on another register and measure it, without disturbing the state more than a negligible amount. As a result, the verification checks can be passed while retaining the ability to decrypt. We note that all existing constructions of encryption with SKL can be broken with similar collusion attacks.

The reason our scheme does not run into such an attack is because we rely on the notion of Attribute-Based Encryption (ABE), which enables exponentially-many secret-keys for every public-key. Consequently, different decryption-keys can be generated as superpositions of different ABE secret-keys. However, it is not clear how this intuition alone can be used to establish security in a provable manner, and we require additional ideas. We now describe these ideas at a high-level.

2.3 Idea Behind the PKE-CR-SKL Scheme

In order to explain the basic idea behind our PKE-CR-SKL scheme, we will first introduce a key-building block, a primitive called SKE-CR-SKL. This is basically a secret-key variant of PKE-CR-SKL, i.e., the setup algorithm only samples a master secret-key $\mathsf{ske.msk}$, and the encryption algorithm encrypts plaintexts under $\mathsf{ske.msk}$. The security requirement is similar. An adversary that receives polynomially many decryption-keys and returns them all, should not be able to distinguish between ciphertexts of different messages. We refer to this security notion as one-time IND-KLA (OT-IND-KLA) security (Definition 4.7). The "one-time" prefix refers to the fact that unlike in PKE-CR-SKL, the adversary does not have the ability to perform chosen plaintext attacks. In other words, the adversary does not see any

[9] In actuality, they use a parallel repetition to amplify security.

ciphertexts before it is required to return its decryption-keys. Although this is a weak security guarantee, it suffices for our PKE-CR-SKL scheme. The description of the PKE-CR-SKL scheme now follows.

The key-generation procedure involves first generating an SKE-CR-SKL decryption-key, which is represented in the computational basis as $\mathsf{ske}.\mathcal{dk} = \sum_u \alpha_u |u\rangle$. Actually, our SKE-CR-SKL scheme needs to satisfy another crucial property, which we call the classical decryption property. This property requires the existence of a classical deterministic algorithm CDec with the following guarantee. For any SKE-CR-SKL ciphertext $\mathsf{ske.ct}$, $\mathsf{CDec}(u, \mathsf{ske.ct})$ correctly decrypts $\mathsf{ske.ct}$ for every string u in the superposition of $\mathsf{ske}.\mathcal{dk}$. Our construction exploits this fact with the help of an ABE scheme as follows.

The actual decryption key is generated as $\mathcal{dk} := \sum_u \alpha_u |u\rangle \otimes |\mathsf{abe.sk}_u\rangle$ where $\mathsf{abe.sk}_u$ is an ABE secret-key corresponding to the key-attribute u. The idea is to now have the encryption algorithm encrypt the plaintext using the ABE scheme, under a carefully chosen ciphertext-policy. Specifically, we wish to embed an SKE-CR-SKL ciphertext $\mathsf{ske.ct}^*$ as part of the policy-circuit, such that the outcome of $\mathsf{CDec}(u, \mathsf{ske.ct}^*)$ determines whether the key $\mathsf{abe.sk}_u$ can decrypt the ABE ciphertext or not. This allows us to consider two ciphertexts $\mathsf{ske.ct}_0^*, \mathsf{ske.ct}_1^*$ of different plaintexts, such that when $\mathsf{ske.ct}_0^*$ is embedded in the policy, then every ABE key $\mathsf{abe.sk}_u$ satisfies the ABE relation. On the other hand, no ABE key satisfies the relation when $\mathsf{ske.ct}_1^*$ is embedded. Observe that in the former case, $\mathcal{dk}$ allows for decryption without disturbing the state (by gentle measurement) while in the latter case, the security of ABE ensures that no adversary can distinguish ciphertexts of different messages[10]. Hence, if we were to undetectably switch the policy-circuit from one corresponding to $\mathsf{ske.ct}_0^*$ to one with $\mathsf{ske.ct}_1^*$, we are done.

While we cannot argue this directly, our main idea is that this switch is undetectable using OT-IND-KLA security, given that all the SKE-CR-SKL keys $\mathsf{ske}.\mathcal{dk}$ are returned. This can be enforced because all the leased decryption keys $\mathcal{dk}$ are required to be returned. Consequently, the verification procedure uncomputes the ABE secret-key register of the returned keys, followed by verifying all the obtained SKE-CR-SKL keys $\mathsf{ske}.\mathcal{dk}'$. However, there is one problem with the template as we have described it so far. This is that both $\mathsf{ske.ct}_0^*, \mathsf{ske.ct}_1^*$ are SKE-CR-SKL ciphertexts, which inherently depend on the master secret-key. In order to remove this dependence and achieve public-key encryption, the actual encryption algorithm uses a dummy ciphertext-policy $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda)$ where $\mathsf{CC.Sim}$ is the simulator of a compute-and-compare obfuscation scheme, a notion we will explain shortly. We will then rely on the security of this obfuscation scheme to switch $\widetilde{C}$ in the security proof, to an appropriate obfuscated circuit with $\mathsf{ske.ct}_0^*$ embedded in it.

In more detail, the ABE scheme allows a key with attribute u to decrypt if and only if the ciphertext policy-circuit $\widetilde{C}$ satisfies $\widetilde{C}(u) = \bot$. Observe that in the construction, $\widetilde{C}$ is generated as $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda)$, which outputs $\bot$ on every input u. Consider now a circuit $\widetilde{C}^*$ that is an obfuscation of the circuit $\mathbf{CC}[D[\mathsf{ske.ct}_0^*], \mathsf{lock}, 0]$ which is described as follows:

Description of $\mathbf{CC}[D[\mathsf{ske.ct}_0^*], \mathsf{lock}, 0]$:

- $\mathsf{ske.ct}_0^*$ is an SKE-CR-SKL encryption of the (dummy) message 0^λ.
- $D[\mathsf{ske.ct}_0^*]$ is a circuit with $\mathsf{ske.ct}_0^*$ hardwired, defined as $D[\mathsf{ske.ct}_0^*](x) = \mathsf{CDec}(x, \mathsf{ske.ct}_0^*)$.
- lock is a value chosen uniformly at random, independently of all other values.
- On input x, the circuit outputs 0 if $D[\mathsf{ske.ct}_0^*](x) = \mathsf{lock}$. Otherwise, it outputs $\bot$.

The above circuit belongs to a sub-class of circuits known as compute-and-compare circuits. Recall that our goal was to avoid the use of IO. We can get away with using IO for this sub-class of circuits, as these so-called compute-and-compare obfuscation schemes are known from LWE [GKW17, WZ17].

The security of the obfuscation can now be used to argue that the switch from $\widetilde{C}$ to $\widetilde{C}^*$ is indistinguishable. Note that to invoke this security guarantee, lock must be a uniform value that is independent of all other values. Next, we can rely on the OT-IND-KLA security of SKE-CR-SKL to switch the ciphertext $\mathsf{ske.ct}_0^*$ embedded in the circuit D to some other ciphertext $\mathsf{ske.ct}_1^*$. The switch would be indistinguishable given that the SKE-CR-SKL decryption-keys are revoked, which we can enforce as mentioned previously. Crucially, we will generate $\mathsf{ske.ct}_1^*$ as an encryption of the value lock corresponding to the above compute-and-compare circuit.[11] This ensures that for every attribute u in the superposition of an SKE-CR-SKL decryption-key $\mathsf{ske}.\mathcal{dk} = \sum_u \alpha_u |u\rangle$, the algorithm $\mathsf{CDec}(u, \mathsf{ske.ct}_1^*)$ outputs the

[10] This requires that the ABE scheme is secure even given superposition access to the key-generation oracle.

[11] Although the compute-and-compare circuit depends on lock in this hybrid, the switch is still justified by OT-IND-KLA security.

value lock. As a consequence, the circuit $\widetilde{C}^*$ will output 0 instead of $\perp$, meaning the key $\mathsf{abe.sk}_u$ does not satisfy the relation in this hybrid, as desired.

It will be clear in the next subsection that our SKE-CR-SKL scheme is implied by OWFs. Since compute-and-compare obfuscation is known from LWE [GKW17, WZ17], and so is Attribute-Based Encryption for polynomial-size circuits [BGG+14][12], we have the following theorem:

Theorem 2.1. *There exists a PKE-CR-SKL scheme satisfying IND-KLA security, assuming the polynomial hardness of the LWE assumption.*

We also observe that using similar ideas as in PKE-CR-SKL, one can obtain an analogous notion of selectively-secure ABE with collusion-resistant secure-key leasing (ABE-CR-SKL) for arbitrary polynomial-time computable circuits. Intuitively, this primitive allows the adversary to declare a target ciphertext-attribute and then make arbitrarily many key-queries adaptively, even ones which satisfy the ABE relation. Then, as long as the adversary verifiably returns all the keys that satisfy the relation, it must lose the ability to decrypt. This is captured formally in the notion of selective IND-KLA security (See the full version [KNP25]). To realize this primitive, we introduce a notion called secret-key functional-encryption with collusion-resistant secure key-leasing (SKFE-CR-SKL), which is a functional encryption analogue of SKE-CR-SKL. From this primitive, we require a notion called selective single-ciphertext security, that is similar to the OT-IND-KLA security of SKE-CR-SKL. Like SKE-CR-SKL, we observe that SKFE-CR-SKL is also implied by OWFs. The other elements of the ABE-CR-SKL construction are the same as in PKE-CR-SKL, namely compute-and-compare obfuscation and an ABE scheme. Consequently, we have the following theorem:

Theorem 2.2. *There exists an ABE-CR-SKL scheme satisfying selective IND-KLA security, assuming the polynomial hardness of the LWE assumption.*

2.4 Constructing SKE-CR-SKL

Next, we will describe how we realize the aforementioned building-block of SKE-CR-SKL satisfying the classical decryption property (Definition 4.5). Our construction will make use of a BB84-based secret-key encryption with certified-deletion (SKE-CD) scheme, a brief description of which is as follows. This is an encryption scheme where the ciphertexts are quantum BB84 states [Wie83, BB20]. Given such a ciphertext, an adversary is later asked to provide a certificate of deletion. If a certificate is provided and verified to be correct, it is guaranteed that the adversary learns nothing about the plaintext even if the secret-key is later revealed. Crucially, we require that the SKE-CD scheme also satisfies a classical decryption property (Definition 3.18). Intuitively, this requires that if the ciphertext is of the form $\mathsf{skecd}.ct = \sum_u \alpha_u |u\rangle$, then every string u in the superposition can be used to decrypt correctly. Specifically, there exists a classical deterministic algorithm $\mathsf{SKECD.CDec}$ such that $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u)$ correctly decrypts $\mathsf{skecd}.ct$, where $\mathsf{skecd.sk}$ is the secret-key.

Let us now recall the functionality offered by SKE-CR-SKL. The encryption algorithm Enc takes as input a master secret-key $\mathsf{ske.msk}$ and a plaintext m and outputs a classical ciphertext $\mathsf{ske.ct}$. The key-generation algorithm $\mathcal{KG}$ takes as input $\mathsf{ske.msk}$ and produces a quantum decryption key $\mathsf{ske}.dk$ along with a corresponding verification-key $\mathsf{ske.vk}$. Decryption of $\mathsf{ske.ct}$ can be performed by $\mathcal{D}ec$ using $\mathsf{ske}.dk$ without disturbing the state by more than a negligible amount. Furthermore, an adversary that receives q (unbounded polynomially many) decryption-keys can be asked to return all of them, before which it does not get to see any ciphertext encrypted under $\mathsf{ske.msk}$. Each returned key can be verified using the verification algorithm and the corresponding verification key. If all q keys are verifiably returned, then it is guaranteed that the adversary cannot distinguish a pair of ciphertexts (of different messages) encrypted under $\mathsf{ske.msk}$. This requirement, termed as OT-IND-KLA security, is captured formally in Definition 4.7. The intuition behind the construction is now described as follows:

Let us begin with the simple encryption algorithm. $\mathsf{Enc}(\mathsf{ske.msk}, \mathsf{m})$ produces a classical output $\mathsf{ske.ct} = (\mathsf{skecd.sk}, z := \mathsf{m} \oplus r)$, where $\mathsf{skecd.sk}$ and r are values specified by $\mathsf{ske.msk}$. The value $\mathsf{skecd.sk}$ is a secret key of an SKE-CD scheme SKECD, while r is a string chosen uniformly at random. Clearly, one can retrieve m from $\mathsf{ske.ct}$ given r. Consequently, each decryption key $\mathsf{ske}.dk$ is essentially an SKECD encryption of r. The idea is that the secret-key $\mathsf{skecd.sk}$ can be obtained from $\mathsf{ske.ct}$, which can then be used to retrieve r from $\mathsf{ske}.dk$. As a consequence,

[12]We show that their ABE scheme is secure even with superposition access to the key-generation oracle.

collusion-resistance (OT-IND-KLA security) can be argued based on the security of SKECD by utilizing the following observations:

- Each decryption-key contains an SKECD encryption of r using independent randomness.
- The adversary must return all the decryption keys (containing SKECD ciphertexts) before it receives the challenge ciphertext (containing the SKECD secret-key).

Furthermore, since $\mathsf{ske}.\mathcal{dk}$ is essentially an SKECD ciphertext, the classical decryption property of SKE-CR-SKL follows easily from the analogous classical decryption property of BB84-based SKE-CD (Definition 3.18).

2.5 Handling Verification Queries

In our previous discussion about the idea behind the PKE-CR-SKL scheme, we left out some details regarding the following. We did not discuss how the key-generation oracle of the ABE scheme can be used to simulate the adversary's view, in the hybrid where $\mathsf{ske.ct}_1^*$ is embedded in the circuit $\widetilde{C}$. Firstly, we note that the ABE scheme can handle superposition key-queries, which we establish by a straightforward argument about the LWE-based ABE scheme of Boneh et al. [BGG+14]. Recall now that in this hybrid, for every leased decryption-key $\mathcal{dk} = \sum_u \alpha_u |u\rangle \otimes |\mathsf{abe.sk}_u\rangle$, each $\mathsf{abe.sk}_u$ can be obtained (in superposition) by querying $\mathsf{ske}.\mathcal{dk} = \sum_u \alpha_u |u\rangle$ to the oracle of ABE. However, we observe that responses to verification queries made by the adversary are not so straightforward to simulate. This is because for each verification query, the ABE secret-key register needs to be uncomputed, for which we will again rely on the ABE key-generation oracle. Specifically, the problem is that for a malformed key $\widetilde{\mathcal{dk}}$, there may exist some $\widetilde{u}$ in the superposition which actually satisfies the ABE relation. Recall that by definition of this relation, it follows that $\mathsf{CDec}(\widetilde{u}, \mathsf{ske.ct}_1^*)$ incorrectly decrypts the ciphertext $\mathsf{ske.ct}_1^*$. To fix this issue, we upgrade our SKE-CR-SKL scheme to satisfy another property called key-testability. This involves the existence of a classical algorithm KeyTest which accepts or rejects. The property requires that when KeyTest is applied in superposition followed by post-selecting on it accepting, every $\widetilde{u}$ in the superposition decrypts correctly. As a result, we are able to apply this key-testing procedure to the register of the SKE-CR-SKL key, followed by simulating the adversary's view using the ABE oracle. Note that we will now consider $\mathcal{KG}(\mathsf{ske.msk})$ to output an additional testing-key $\mathsf{ske.tk}$ along with $\mathsf{ske}.\mathcal{dk}$ and $\mathsf{ske.vk}$. The key-testability requirements are specified in more detail as follows:

- **Security:** There exists an algorithm KeyTest such that no QPT adversary can produce a classical value dk and a message m such that:
  - $\mathsf{CDec}(\mathsf{dk}, \mathsf{ske.ct}) \neq \mathsf{m}$, where $\mathsf{ske.ct} \leftarrow \mathsf{Enc}(\mathsf{ske.msk}, \mathsf{m})$.
  - $\mathsf{KeyTest}(\mathsf{ske.tk}, \mathsf{dk}) = 1$.
- **Correctness:** If KeyTest is applied in superposition to $\mathsf{ske}.\mathcal{dk}$ and the output is measured to obtain outcome 1, $\mathsf{ske}.\mathcal{dk}$ should be almost undisturbed.

2.6 Upgrading SKE-CR-SKL with Key-Testability

We will now explain how the SKE-CR-SKL construction is modified to satisfy the aforementioned key-testability property. First, we mention a classical decryption property of SKECD (Definition 3.18) that we crucially rely on. A ciphertext $\mathsf{skecd}.ct$ (corresponding to some message m) of SKECD is essentially a BB84 state $|x\rangle_\theta$. The property guarantees the existence of an algorithm SKECD.CDec such that for any string u that matches x at all computational basis positions specified by θ, $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u) = \mathsf{m}$ with overwhelming probability. Recall now that in our SKE-CR-SKL construction, ciphertexts are of the form $\mathsf{ske.ct} = (\mathsf{skecd.sk}, z = r \oplus \mathsf{m})$ and decryption-keys are essentially SKECD encryptions of the value r. Consequently, the algorithm CDec of SKE-CR-SKL works as follows:

$\underline{\mathsf{CDec}(u, \mathsf{ske.ct})}$:

- Parse $\mathsf{ske.ct} = (\mathsf{skecd.sk}, z)$.

- Output $z \oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u)$.

As a result, it is sufficient for us to ensure that no QPT adversary can output a value dk such that $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \mathsf{dk})$ produces a value different from r. By the aforementioned classical-decryption property of SKECD, it suffices to bind the adversary to the computational basis values of a ciphertext $\mathsf{skecd}.ct = |x\rangle_\theta$. For this, we employ a technique reminiscent of the Lamport-signature scheme [Lam79]. Thereby, an additional "signature" register that is entangled with the SKECD ciphertext $\mathsf{skecd}.ct = |x\rangle_\theta$ is utilized. We note that similar techniques for signing BB84 states were employed in previous works on certified deletion [HKM+24, KNY23]. Specifically, let $\mathsf{SKECD.CT}_i$ denote the register holding the i-th qubit of $\mathsf{skecd}.ct$ and $s_{i,0}, s_{i,1}$ be randomly chosen pre-images from the domain of an OWF f. Then, the following map is performed on registers $\mathsf{SKECD.CT}_i$ and S_i where the latter is initialized to $|0\dots0\rangle$:

$$|u_i\rangle_{\mathsf{SKECD.CT}_i} \otimes |v_i\rangle_{\mathsf{S}_i} \to |u_i\rangle_{\mathsf{SKECD.CT}_i} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S}_i}$$

Let ρ_i be the resulting state. The SKE-CR-SKL decryption-key $\mathsf{ske}.d\mathcal{K}$ is set to be the state $\rho_1 \otimes \dots \otimes \rho_{\ell_{\mathsf{ct}}}$ where ℓ_{ct} is the length of $\mathsf{skecd}.ct$. Thereby, the testing-key $\mathsf{ske.tk}$ will consist of the values $f(s_{i,0}), f(s_{i,1})$ for each $i \in [\ell_{\mathsf{ct}}]$. Observe now that for a returned (possibly altered) decryption-key $\widetilde{d\mathcal{K}}$ (or $\mathsf{ske}.\widetilde{d\mathcal{K}}$), it is possible to check for each qubit whether the superposition term u_i is associated with the correct pre-image s_{i,u_i}. This can be done by forward evaluating the pre-image register and comparing it with the value $f(s_{i,u_i})$ that is specified in $\mathsf{ske.tk}$. This is essentially the KeyTest algorithm. It is easy to see that this procedure does not disturb the state when applied to the unaltered decryption key $d\mathcal{K}$ (or $\mathsf{ske}.d\mathcal{K}$). Moreover, observe that the adversary does not receive the pre-image $s_{i,1-x[i]}$ for any computational basis position i. Consequently, we show that the adversary cannot produce a value dk whose computational basis values are inconsistent with those of x, unless it can invert outputs of f. From the previous discussion, it follows that if the computational basis values of dk are consistent with x, then dk cannot result in the incorrect decryption of $\mathsf{ske.ct}$.

3 Preliminaries

Notations and conventions. In this paper, standard math or sans serif font stands for classical algorithms (e.g., C or Gen) and classical variables (e.g., x or pk). Calligraphic font stands for quantum algorithms (e.g., $\mathcal{Gen}$) and calligraphic font and/or the bracket notation for (mixed) quantum states (e.g., q or $|\psi\rangle$).

Let $[\ell]$ denote the set of integers $\{1, \cdots, \ell\}$, λ denote a security parameter, and $y := z$ denote that y is set, defined, or substituted by z. For a finite set X and a distribution D, $x \leftarrow X$ denotes selecting an element from X uniformly at random, and $x \leftarrow D$ denotes sampling an element x according to D. Let $y \leftarrow \mathsf{A}(x)$ and $y \leftarrow \mathcal{A}(\chi)$ denote assigning to y the output of a probabilistic or deterministic algorithm A and a quantum algorithm $\mathcal{A}$ on an input x and χ, respectively. PPT and QPT algorithms stand for probabilistic polynomial-time algorithms and polynomial-time quantum algorithms, respectively. Let negl denote a negligible function. For strings $x, y \in \{0,1\}^n$, $x \cdot y$ denotes $\bigoplus_{i\in[n]} x_i y_i$ where x_i and y_i denote the ith bit of x and y, respectively. For random variables X and Y, we use the notation $X \approx Y$ to denote that these are computationally indistinguishable. Likewise, $X \approx_s Y$ denotes that they are statistically indistinguishable.

3.1 One Way to Hiding Lemmas

Lemma 3.1 (O2H Lemma [AHU19]). *Let $G, H : X \to Y$ be any functions, z be a random value, and $S \subseteq X$ be a random set such that $G(x) = H(x)$ holds for every $x \notin S$. The tuple (G, H, S, z) may have arbitrary joint distribution. Furthermore, let $\mathcal{A}$ be a quantum oracle algorithm that makes at most q quantum queries. Let $\mathcal{B}$ be an algorithm such that $\mathcal{B}^H$ on input z chooses $i \leftarrow [q]$, runs $\mathcal{A}^H(z)$, measures $\mathcal{A}$'s i-th query, and outputs the measurement outcome. Then, we have:*

$$\left|\Pr\left[\mathcal{A}^H(z) = 1\right] - \Pr\left[\mathcal{A}^G(z) = 1\right]\right| \leq 2q \cdot \sqrt{\Pr[\mathcal{B}^H(z) \in S]} \ .$$

We require a generalization of this lemma, where $\mathcal{A}$ receives an additional quantum oracle $\mathcal{Q}$ in both worlds. Consequently, we consider $\mathcal{B}$ to be given oracle access to $\mathcal{Q}$, which it uses to simulate the oracle calls of $\mathcal{A}$ to $\mathcal{Q}$. Notice that if the outputs of $\mathcal{Q}$ were classical, we could have simply defined augmented oracles G' (likewise H') based on

G (likewise H) and $\mathcal{Q}$. However, the oracles $\mathcal{Q}$ we consider will have classical inputs and quantum state outputs. Consequently, the lemma we require is stated as follows:

Lemma 3.2 (O2H Lemma with Auxiliary Quantum Oracle). *Let $G, H : X \to Y$ be any functions, z be a random value, and $S \subseteq X$ be a random set such that $G(x) = H(x)$ holds for every $x \notin S$. The tuple (G, H, S, z) may have arbitrary joint distribution. Furthermore, let $\mathcal{Q}$ be a quantum oracle that is arbitrarily correlated with the tuple (G, H, S, z), takes classical input and produces a (possibly mixed) quantum state as output. Let $\mathcal{A}$ be a quantum oracle algorithm that makes at most q quantum queries to the oracles H or G, and arbitrarily many queries to the oracle $\mathcal{Q}$. Let $\mathcal{B}$ be an algorithm such that $\mathcal{B}^{\mathcal{Q},H}$ on input z, chooses $i \leftarrow [q]$, runs $\mathcal{A}^{\mathcal{Q},H}(z)$, measures $\mathcal{A}$'s i-th query to H, and outputs the measurement outcome. Then, we have:*

$$\left|\Pr\left[\mathcal{A}^{\mathcal{Q},H}(z) = 1\right] - \Pr\left[\mathcal{A}^{\mathcal{Q},G}(z) = 1\right]\right| \leq 2q \cdot \sqrt{\Pr[\mathcal{B}^{\mathcal{Q},H}(z) \in S]} \ .$$

Proof of Lemma 3.2. Let us consider an adversary $\widetilde{\mathcal{A}}$ that receives as input the description $\langle \mathcal{Q} \rangle$ of $\mathcal{Q}$[13], along with the input z used by $\mathcal{A}$. Given oracle access to H (likewise G) and $(z, \langle \mathcal{Q} \rangle)$ as input, $\widetilde{\mathcal{A}}$ simply runs $\mathcal{A}^{H,\mathcal{Q}}(z)$ (likewise $\mathcal{A}^{G,\mathcal{Q}}(z)$) by simulating its queries to $\mathcal{Q}$ using the description $\langle \mathcal{Q} \rangle$. Then, the O2H Lemma (Lemma 3.1) implies the existence of an algorithm $\widetilde{\mathcal{B}}^{H}(z, \langle \mathcal{Q} \rangle)$ that chooses $i \leftarrow [q]$, runs $\widetilde{\mathcal{A}}^{H}(z, \langle \mathcal{Q} \rangle)$, measures its i-th query to H and outputs the measurement outcome. Observe that the algorithms $\widetilde{A}$ and $\widetilde{B}$ do not make use of the description $\langle \mathcal{Q} \rangle$ except for simulating the queries made by $\mathcal{A}$. Consequently, there exists an algorithm $\mathcal{B}^{\mathcal{Q},H}(z)$ equivalent to $\widetilde{\mathcal{B}}^{H}(z, \langle \mathcal{Q} \rangle)$ that directly runs $\mathcal{A}$ (instead of $\widetilde{\mathcal{A}}$) and simulates its oracle queries to $\mathcal{Q}$ using its own access to $\mathcal{Q}$. □

Remark 3.3. We assume that $\mathcal{B}$ also outputs the measured index i. However, this output is not taken into account for notation such as $\mathcal{B}^{\mathcal{Q},H}(z) \in S$ for the sake of simplicity.

3.2 Standard Cryptographic Tools

Attribute-Based Encryption.

Definition 3.4 (Attribute-Based Encryption). *An ABE scheme* ABE *is a tuple of four PPT algorithms* $(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$. *Below, let $\mathcal{X} = \{\mathcal{X}_\lambda\}_\lambda$, $\mathcal{Y} = \{\mathcal{Y}_\lambda\}_\lambda$, and $R = \{R_\lambda : \mathcal{X}_\lambda \times \mathcal{Y}_\lambda \to \{0,1\}\}_\lambda$ be the ciphertext attribute space, key attribute space, and the relation associated with* ABE*, respectively. We note that we will abuse the notation and occasionally drop the subscript for these spaces for notational simplicity. We also note that the message space is set to be $\{0,1\}^\ell$ below.*

$\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter 1^λ and outputs a public key* pk *and master secret key* msk.

$\mathsf{KG}(\mathsf{msk}, y, r) \to \mathsf{sk}_y$**:** *The key generation algorithm* KG *takes a master secret key* msk*, a key attribute $y \in \mathcal{Y}$, and explicit randomness r. It outputs a decryption key* sk_y*. Note that* KG *is deterministic.*[14]

$\mathsf{Enc}(\mathsf{pk}, x, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes a public key* pk*, a ciphertext attribute $x \in \mathcal{X}$, and a message* m*, and outputs a ciphertext* ct.

$\mathsf{Dec}(\mathsf{sk}_y, \mathsf{ct}) \to z$**:** *The decryption algorithm takes a secret key* sk_y *and a ciphertext* ct *and outputs $z \in \{\bot\} \cup \{0,1\}^\ell$.*

Correctness: *We require that*

$$\Pr\left[\mathsf{Dec}(\mathsf{sk}_y, \mathsf{ct}) = \mathsf{m} \ : \ \begin{array}{l}(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda), \\ r \leftarrow \{0,1\}^{\mathrm{poly}(\lambda)}, \\ \mathsf{sk}_y \leftarrow \mathsf{KG}(\mathsf{msk}, y, r), \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{pk}, x, \mathsf{m})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

holds for all $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ such that $R(x, y) = 0$ and $m \in \{0,1\}^\ell$.

[13] The O2H Lemma (Lemma 3.1) holds even if z is exponentially large, so the description of $\mathcal{Q}$ need not be concise.

[14] In the standard syntax, KG does not take explicit randomness, and is probabilistic. This change is just for notational convention in our schemes.

By setting $\mathcal{X}$, $\mathcal{Y}$, and R appropriately, we can recover important classes of ABE. In particular, if we set $\mathcal{X}_\lambda = \mathcal{Y}_\lambda = \{0,1\}^*$ and define R_λ so that $R_\lambda(x,y) = 0$ if $x = y$ and $R_\lambda(x,y) = 1$ otherwise, we recover the definition of identity-based encryption (IBE). If we set $\mathcal{X}_\lambda = \{0,1\}^{n(\lambda)}$ and $\mathcal{Y}_\lambda$ to be the set of all circuits with input space $\{0,1\}^{n(\lambda)}$ and size at most $s(\lambda)$, where n and s are some polynomials, and define R so that $R(x,y) = y(x)$, we recover the definition of (key policy) ABE for circuits.

We introduce a new security notion for ABE that we call quantum selective-security for ABE where the adversary is allowed to get access to the key generation oracle in super-position.

Definition 3.5 (Quantum Selective-Security for ABE). *We say that* ABE *is a* selective-secure *ABE scheme for relation* $R : \mathcal{X} \times \mathcal{Y} \to \{0,1\}$*, if it satisfies the following requirement, formalized by the experiment* $\mathsf{Exp}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger* $\mathcal{Ch}$*:*

1. $\mathcal{A}$ *declares the challenge ciphertext attribute* x^**.* $\mathcal{Ch}$ *runs* $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ *and sends* pk *to* $\mathcal{A}$*.*
2. $\mathcal{A}$ *can get access to the following quantum key generation oracle.*

   $O_{\mathtt{qkg}}(\mathsf{Y}, \mathsf{Z})$**:** *Given registers* Y *and* Z*, it first applies the map* $|y\rangle_\mathsf{Y} |b\rangle_\mathsf{B} \to |y\rangle_\mathsf{Y} |b \oplus R(x^*, y)\rangle_\mathsf{B}$ *and measures the register* B*, where* B *is initialized to* $|0\rangle_\mathsf{B}$*. If the result is* 0*, it returns* $\perp$*. Otherwise, it chooses* $r \leftarrow \{0,1\}^{\mathrm{poly}(\lambda)}$*, applies the map* $|y\rangle_\mathsf{Y} |z\rangle_\mathsf{Z} \to |y\rangle_\mathsf{Y} |z \oplus \mathsf{KG}(\mathsf{msk}, y, r)\rangle_\mathsf{Z}$ *and returns the registers* Y *and* Z*.*
3. *At some point,* $\mathcal{A}$ *sends* $(\mathsf{m}_0, \mathsf{m}_1)$ *to* $\mathcal{Ch}$*. Then,* $\mathcal{Ch}$ *generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{pk}, x^*, \mathsf{m}_\mathsf{coin})$ *and sends* ct^* *to* $\mathcal{A}$*.*
4. *Again,* $\mathcal{A}$ *can get access to the oracle* $O_{\mathtt{qkg}}$*.*
5. $\mathcal{A}$ *outputs a guess* coin' *for* coin *and the experiment outputs* coin'*.*

We say that ABE *satisfies quantum selective security if, for all QPT* $\mathcal{A}$*, it holds that*

$$\mathsf{Adv}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

Boneh and Zhandry [BZ13b] introduced a similar quantum security notion for IBE and argued that it is straightforward to prove the quantum security of the IBE scheme by [ABB10], by leveraging the lattice trapdoor based proof technique. It is easy to prove the quantum selective security of the ABE scheme for circuits by Boneh et al. [BGG+14], which relies on the lattice trapdoor based proof technique as well. Formally, we have the following theorem.

Theorem 3.6. *Assuming the hardness of the LWE problem, there exists a quantum selectively secure ABE scheme for all relations computable in polynomial time.*

We elaborate on this in Appendix A.

We also focus on the IBE setting and use the standard adaptive-security for IBE in Section 9.

Definition 3.7 (Adaptive-Security for IBE). *We say that* IBE *is an* adaptive-secure *IBE scheme for if it satisfies the following requirement, formalized by the experiment* $\mathsf{Exp}^{\mathsf{ada\text{-}ind}}_{\mathsf{IBE},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger* $\mathcal{Ch}$*:*

1. $\mathcal{Ch}$ *runs* $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ *and sends* pk *to* $\mathcal{A}$*.*
2. $\mathcal{A}$ *can get access to the following key generation oracle.*

   $O_{\mathtt{kg}}(\mathsf{id})$**:** *Given an identity* id*, generates* $\mathsf{sk}_\mathsf{id} \leftarrow \mathsf{IBE.KG}(\mathsf{msk}, \mathsf{id})$*, and returns* sk_id *to* $\mathcal{A}$*.*
3. *At some point,* $\mathcal{A}$ *sends* $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$ *to* $\mathcal{Ch}$*. Then,* $\mathcal{Ch}$ *generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{pk}, \mathsf{id}^*, \mathsf{m}_\mathsf{coin})$ *and sends* ct^* *to* $\mathcal{A}$*.*
4. *Again,* $\mathcal{A}$ *can get access to the oracle* $O_{\mathtt{kg}}$*. However,* $O_{\mathtt{kg}}$ *does not answer to* id^**.*

5. *$\mathcal{A}$ outputs a guess* coin' *for* coin *and the experiment outputs* coin'.

We say that IBE *satisfies adaptive security if, for all QPT $\mathcal{A}$, it holds that*

$$\mathsf{Adv}^{\mathsf{ada\text{-}ind}}_{\mathsf{IBE},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind}}_{\mathsf{IBE},\mathcal{A}}(1^\lambda,0)\to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind}}_{\mathsf{IBE},\mathcal{A}}(1^\lambda,1)\to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

Theorem 3.8 ([ABB10]). *Assuming the polynomial hardness of the LWE problem, there exists an adaptively secure IBE scheme.*

Compute-and-Compare Obfuscation. We define a class of circuits called compute-and-compare circuits as follows:

Definition 3.9 (Compute-and-Compare Circuits). *A compute-and-compare circuit* $\mathbf{CC}[P,\mathsf{lock},\mathsf{m}]$ *is of the form*

$$\mathbf{CC}[P,\mathsf{lock},\mathsf{m}](x)\begin{cases}\mathsf{m} & \text{if } P(x)=\mathsf{lock}\\ \bot & \text{otherwise}\end{cases}$$

where P is a circuit, lock *is a string called the lock value, and* m *is a message.*

We now introduce the definition of compute-and-compare obfuscation. We assume that a program P has an associated set of parameters pp_P (input size, output size, circuit size) which we do not need to hide.

Definition 3.10 (Compute-and-Compare Obfuscation). *A PPT algorithm* $\mathsf{CC.Obf}$ *is an obfuscator for the family of distributions $D=\{D_\lambda\}$ if the following holds:*

Functionality Preserving: *There exists a negligible function* negl *such that for all programs P, all lock values* lock*, and all messages* m*, it holds that*

$$\Pr\left[\forall x, \widetilde{P}(x)=\mathbf{CC}[P,\mathsf{lock},\mathsf{m}](x) : \widetilde{P}\leftarrow \mathsf{CC.Obf}(1^\lambda,P,\mathsf{lock},\mathsf{m})\right]\geq 1-\mathsf{negl}(\lambda).$$

Distributional Indistinguishability: *There exists an efficient simulator* Sim *such that for all messages* m*, we have*

$$(\mathsf{CC.Obf}(1^\lambda,P,\mathsf{lock},\mathsf{m}),aux)\approx(\mathsf{CC.Sim}(1^\lambda,\mathsf{pp}_P,|\mathsf{m}|),aux),$$

where $(P,\mathsf{lock},aux)\leftarrow D_\lambda$.

Theorem 3.11 ([GKW17, WZ17]). *If the LWE assumption holds, there exists compute-and-compare obfuscation for all families of distributions $D=\{D_\lambda\}$, where each D_λ outputs uniformly random lock value* lock *independent of P and aux.*

Pseudorandom Functions.

Definition 3.12 (Pseudorandom Function). *Let* $\{\mathsf{PRF}_\mathsf{K} : \{0,1\}^{\ell_1}\to\{0,1\}^{\ell_2} \mid \mathsf{K}\in\{0,1\}^\lambda\}$ *be a family of polynomially computable functions, where ℓ_1 and ℓ_2 are some polynomials of λ. We say that* PRF *is a pseudorandom function (PRF) family if, for any QPT distinguisher $\mathcal{A}$, there exists* $\mathsf{negl}(\cdot)$ *such that it holds that*

$$\left|\Pr\left[\mathcal{A}^{\mathsf{PRF}_\mathsf{K}(\cdot)}(1^\lambda)\to 1 \mid \mathsf{K}\leftarrow\{0,1\}^\lambda\right]-\Pr\left[\mathcal{A}^{\mathsf{R}(\cdot)}(1^\lambda)\to 1 \mid \mathsf{R}\leftarrow\mathcal{U}\right]\right|\leq\mathsf{negl}(\lambda),$$

where $\mathcal{U}$ is the set of all functions from $\{0,1\}^{\ell_1}$ to $\{0,1\}^{\ell_2}$.

Theorem 3.13 ([GGM86]). *If one-way functions exist, then for all efficiently computable functions $n(\lambda)$ and $m(\lambda)$, there exists a PRF that maps $n(\lambda)$ bits to $m(\lambda)$ bits.*

Garbling schemes.

Definition 3.14 (Garbling schemes). *A garbling scheme* GC *is a tuple of PPT algorithms* $\mathsf{GC} = (\mathsf{Grbl}, \mathsf{GCEval})$.

$\mathsf{Grbl}(1^\lambda, C) \to (\{\mathsf{lab}_{i,b}\}_{i\in[\ell],b\in\{0,1\}}, \widetilde{C})$**:** *The garbling algorithm takes a security parameter* 1^λ *and a circuit* C *and outputs labels* $\{\mathsf{lab}_{i,b}\}_{i\in[\ell],b\in\{0,1\}}$ *and garbled version of the circuit* $\widetilde{C}$*, where* ℓ *is the input length of* C.

$\mathsf{GCEval}(\widetilde{C}, \{\mathsf{lab}_i\}_{i\in[\ell]}) \to z$**:** *The evaluation algorithm* GCEval *takes the garbled circuit* $\widetilde{C}$ *and labels* $\{\mathsf{lab}_i\}_{i\in[\ell]}$ *and outputs an evaluation result* z.

Correctness: *We require that*

$$\Pr\left[\mathsf{GCEval}(\widetilde{C}, \{\mathsf{lab}_{i,x_i}\}_{i\in[\ell]}) = C(x) \;\middle|\; \mathsf{Grbl}(1^\lambda, C)) \to (\{\mathsf{lab}_{i,b}\}_{i\in[\ell],b\in\{0,1\}}, \widetilde{C})\right] = 1 - \mathsf{negl}(\lambda)$$

holds for all $\ell \in \mathbb{N}$, $x \in \{0,1\}^\ell$ *and* C *with input length* ℓ*, where* x_i *is the* i*-th bit of* x.

Security: *We require that there exists a PPT algorithm* $\mathsf{Sim.GC}$ *such the following distributions are computationally indistinguishable for all* $\ell \in \mathbb{N}$, $x \in \{0,1\}^\ell$*, and circuit* C *with input length* ℓ*:*

$$(\{\mathsf{lab}_{i,x_i}\}_{i\in[\ell]}, \widetilde{C}) \approx_c \mathsf{Sim.GC}(1^\lambda, \mathsf{info}(C), C(x))$$

where $\mathsf{Grbl}(1^\lambda, C)) \to (\{\mathsf{lab}_{i,b}\}_{i\in[\ell],b\in\{0,1\}}, \widetilde{C})$ *and* $\mathsf{info}(C)$ *refers to the size of* C*, input and output lengths of* C.

We note that we will drop $\mathsf{info}(C)$ *from the inputs to* $\mathsf{Sim.GC}$ *when it is clear from the context.*

Theorem 3.15 ([Yao86, LP09]). *If there exists a one-way function, there exists secure garbling scheme.*

Certified Deletion. We present the definitions for SKE with certified deletion.

Definition 3.16 (SKE-CD (Syntax)). *An SKE-CD scheme is a tuple of algorithms* $(\mathsf{KG}, \mathcal{Enc}, \mathcal{Dec}, \mathcal{Del}, \mathsf{Vrfy})$ *with plaintext space* $\mathcal{M}$ *and key space* $\mathcal{K}$.

$\mathsf{KG}(1^\lambda) \to \mathsf{sk}$**:** *The key generation algorithm takes as input the security parameter* 1^λ *and outputs a secret key* $\mathsf{sk} \in \mathcal{K}$.

$\mathcal{Enc}(\mathsf{sk}, \mathsf{m}) \to (ct, \mathsf{vk})$**:** *The encryption algorithm takes as input* sk *and a plaintext* $\mathsf{m} \in \mathcal{M}$ *and outputs a ciphertext* ct *and a verification key* vk.

$\mathcal{Dec}(\mathsf{sk}, ct) \to \mathsf{m}'$**:** *The decryption algorithm takes as input* sk *and* ct *and outputs a plaintext* $\mathsf{m}' \in \mathcal{M}$ *or* $\perp$.

$\mathcal{Del}(ct) \to \mathsf{cert}$**:** *The deletion algorithm takes as input* ct *and outputs a certificate* cert.

$\mathsf{Vrfy}(\mathsf{vk}, \mathsf{cert}) \to \top/\perp$**:** *The verification algorithm takes* vk *and* cert *as input and outputs* $\top$ *or* $\perp$.

Decryption correctness: *There exists a negligible function* negl *such that for any* $\mathsf{m} \in \mathcal{M}$,

$$\Pr\left[\mathcal{Dec}(\mathsf{sk}, ct) = \mathsf{m} \;:\; \begin{array}{l}\mathsf{sk} \leftarrow \mathsf{KG}(1^\lambda)\\ (ct, \mathsf{vk}) \leftarrow \mathcal{Enc}(\mathsf{sk}, \mathsf{m})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *There exists a negligible function* negl *such that for any* $\mathsf{m} \in \mathcal{M}$,

$$\Pr\left[\mathsf{Vrfy}(\mathsf{vk}, \mathsf{cert}) = \top \;:\; \begin{array}{l}\mathsf{sk} \leftarrow \mathsf{KG}(1^\lambda)\\ (ct, \mathsf{vk}) \leftarrow \mathcal{Enc}(\mathsf{sk}, \mathsf{m})\\ \mathsf{cert} \leftarrow \mathcal{Del}(ct)\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

We now introduce indistinguishability against Chosen Verification Attacks (CVA).

Definition 3.17 (IND-CVA-CD Security). *We consider the following security experiment* $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,\mathsf{coin})$.

1. *The challenger computes* $\mathsf{sk}\leftarrow\mathsf{KG}(1^\lambda)$.
2. *Thoughout the experiment,* $\mathcal{A}$ *can get access to the following oracle.*

   $O_{\mathcal{E}nc}(\mathsf{m})$**:** *On input* m*, it generates* $(ct,\mathsf{vk})\leftarrow\mathcal{E}nc(\mathsf{sk},\mathsf{m})$ *and returns* (vk,ct).
3. $\mathcal{A}$ *sends* $(\mathsf{m}_0,\mathsf{m}_1)\in\mathcal{M}^2$ *to the challenger.*
4. *The challenger computes* $(ct^*,\mathsf{vk}^*)\leftarrow\mathcal{E}nc(\mathsf{sk},\mathsf{m}_{\mathsf{coin}})$ *and sends* ct^* *to* $\mathcal{A}$.
5. *Hereafter,* $\mathcal{A}$ *can get access to the following oracle, where* V *is initialized to* $\bot$.

   $O_{\mathsf{Vrfy}}(\mathsf{cert})$**:** *On input* cert*, it returns* sk *and updates* V *to* $\top$ *if* $\mathsf{Vrfy}(\mathsf{vk}^*,\mathsf{cert})=\top$. *Otherwise, it returns* $\bot$.
6. *When* $\mathcal{A}$ *outputs* $\mathsf{coin}'\in\{0,1\}$*, the experiment outputs* coin' *if* $V=\top$ *and otherwise outputs* 0.

We say that SKECD *is IND-CVA-CD secure if for any QPT* $\mathcal{A}$*, it holds that*

$$\mathsf{Adv}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda):=\left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,0)=1\right]-\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,1)=1\right]\right|\\ \leq\mathsf{negl}(\lambda).$$

Definition 3.18 (BB84-Based SKE-CD). *We say that an SKE-CD scheme* $\mathsf{SKECD}=(\mathsf{KG},\mathcal{E}nc,\mathcal{D}ec,\mathcal{D}el,\mathsf{Vrfy})$ *is a BB84-based SKE-CD scheme if it satisfies the following conditions.*

- *Let* $(ct,\mathsf{vk})\leftarrow\mathcal{E}nc(\mathsf{sk},\mathsf{m})$. vk *is of the form* $(x,\theta)\in\{0,1\}^{\ell_{\mathsf{ct}}}\times\{0,1\}^{\ell_{\mathsf{ct}}}$*, and* ct *is of the form* $|\psi_1\rangle\otimes\cdots\otimes|\psi_{\ell_{\mathsf{ct}}}\rangle$*, where*

$$|\psi_i\rangle=\begin{cases}|x[i]\rangle & if\ \ \theta[i]=0\\ |0\rangle+(-1)^{x[i]}|1\rangle & if\ \ \theta[i]=1.\end{cases}$$

  Moreover, there exists $n<\ell_{\mathsf{ct}}$ *such that* $\theta[i]=0$ *for every* $i\in[n+1,\ell_{\mathsf{ct}}]$*. We call* $x[n+1]\|\cdots\|x[\ell_{\mathsf{ct}}]$ *a classical part of* ct*. The parameter* n *is specified by a construction. The classical part has information of* θ*, and we can compute* θ *from it and* sk.

- $\mathcal{D}el(ct)$ *measures each qubit of* ct *in the Hadamard basis and outputs the measurement result* $\mathsf{cert}\in\{0,1\}^{\ell_{\mathsf{ct}}}$.
- $\mathsf{Vrfy}(\mathsf{vk},\mathsf{cert})$ *outputs* $\top$ *if* $\mathsf{cert}[i]=x[i]$ *holds for every* $i\in[n]$ *such that* $\theta[i]=1$*, and* 0 *otherwise.*
- ***Classical Decryption Property:*** *There exists an additional deterministic polynomial time algorithm* CDec *with the following property. Let* $(ct,\mathsf{vk})\leftarrow\mathcal{E}nc(\mathsf{sk},\mathsf{m})$*, where* $\mathsf{vk}=(x,\theta)$*. Let* $u\in\{0,1\}^{\ell_{\mathsf{ct}}}$ *be any string such that* $u[i]=x[i]$ *for all* $i:\theta[i]=0$*. Then, the following holds:*

$$\Pr\left[\mathsf{CDec}(\mathsf{sk},u)=\mathsf{m}\right]\geq1-\mathsf{negl}(\lambda)$$

Theorem 3.19. *There exists a BB84-based SKE-CD scheme satisfying IND-CVA-CD security, assuming the existence of a CPA-secure Secret-Key Encryption scheme.*

Kitagawa and Nishimaki [KN22] claimed the same statement as Theorem 3.19. However, their proof has a gap because known BB84-based SKE-CD schemes do not satisfy the unique certificate property, which they introduced. Hence, we prove Theorem 3.19 in the full version [KNP25].

4 Encryption with Collusion-Resistant SKL

In this section, we define the notions of public-key and secret-key encryption with collusion-resistant secure key-leasing.

4.1 Definitions of PKE-CR-SKL

The syntax of PKE-CR-SKL is defined as follows.

Definition 4.1 (PKE-CR-SKL). *A PKE-CR-SKL scheme* $\mathsf{PKE\text{-}CR\text{-}SKL}$ *is a tuple of five algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$. *Below, let* $\mathcal{M}$ *be the message space of* $\mathsf{PKE\text{-}CR\text{-}SKL}$.

$\mathsf{Setup}(1^\lambda) \to (\mathsf{ek}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter* 1^λ*, and outputs an encryption key* ek *and a master secret-key* msk.

$\mathcal{KG}(\mathsf{msk}) \to (\mathit{dk}, \mathsf{vk})$**:** *The key generation algorithm takes the master secret-key* msk *as input, and outputs a decryption key* dk *and a verification key* vk.

$\mathsf{Enc}(\mathsf{ek}, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes an encryption key* ek *and a message* $\mathsf{m} \in \mathcal{M}$*, and outputs a ciphertext* ct.

$\mathcal{Dec}(\mathit{dk}, \mathsf{ct}) \to \widetilde{\mathsf{m}}/\bot$**:** *The decryption algorithm takes a decryption key* dk *and a ciphertext* ct*, and outputs a value* $\widetilde{\mathsf{m}}$ *or* $\bot$.

$\mathcal{Vrfy}(\mathsf{vk}, \widetilde{\mathit{dk}}) \to \top/\bot$**:** *The verification algorithm takes a verification key* vk *and a (possibly malformed) decryption key* $\widetilde{\mathit{dk}}$*, and outputs* $\top$ *or* $\bot$.

Decryption correctness: *For every* $\mathsf{m} \in \mathcal{M}$*, we have*

$$\Pr\left[\mathcal{Dec}(\mathit{dk}, \mathsf{ct}) = \mathsf{m} \ : \ \begin{array}{l}(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{dk}, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}) \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{ek}, \mathsf{m})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *We have*

$$\Pr\left[\mathcal{Vrfy}(\mathsf{vk}, \mathit{dk}) = \top \ : \ \begin{array}{l}(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{dk}, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Remark 4.2. We can assume without loss of generality that a decryption key of a PKE-CR-SKL scheme is reusable, i.e., it can be reused to decrypt (polynomially) many ciphertexts. In particular, we can assume that for honestly generated ct and dk, if we decrypt ct by using dk, the state of the decryption key after the decryption is negligibly close to that before the decryption in terms of trace distance. This is because the output of the decryption is almost deterministic by decryption correctness, and thus such an operation can be done without almost disturbing the input state by the gentle measurement lemma [Win99].

Definition 4.3 (IND-KLA Security). *We say that a PKE-CR-SKL scheme* $\mathsf{PKE\text{-}CR\text{-}SKL}$ *with the message space* $\mathcal{M}$ *is IND-KLA secure, if it satisfies the following requirement, formalized by the experiment* $\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger* $\mathcal{Ch}$*:*

1. $\mathcal{Ch}$ *runs* $(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ *and sends* ek *to* $\mathcal{A}$.
2. $\mathcal{A}$ *requests* q *decryption keys for some polynomial* q*.* $\mathcal{Ch}$ *generates* $(\mathit{dk}_i, \mathsf{vk}_i) \leftarrow \mathcal{KG}(\mathsf{msk})$ *for every* $i \in [q]$ *and sends* $\mathit{dk}_1, \ldots, \mathit{dk}_q$ *to* $\mathcal{A}$.
3. $\mathcal{A}$ *can get access to the following (stateful) verification oracle* $O_{\mathcal{Vrfy}}$ *where* V_i *is initialized to* $\bot$ *for all* $i \in [q]$*:*

   $O_{\mathcal{Vrfy}}(i, \widetilde{\mathit{dk}})$**:** *It runs* $d \leftarrow \mathcal{Vrfy}(\mathsf{vk}_i, \widetilde{\mathit{dk}})$ *and returns* d.
   If $V_i = \bot$ *and* $d = \top$*, it updates* $V_i := \top$.
4. $\mathcal{A}$ *sends* $(\mathsf{m}^*_0, \mathsf{m}^*_1) \in \mathcal{M}^2$ *to the challenger. If* $V_i = \bot$ *for some* $i \in [q]$*, the challenger outputs* 0 *as the final output of this experiment. Otherwise, the challenger generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{ek}, \mathsf{m}^*_{\mathsf{coin}})$ *and sends* ct^* *to* $\mathcal{A}$.

5. *$\mathcal{A}$ outputs a guess* coin' *for* coin. *$\mathcal{C}h$ outputs* coin' *as the final output of the experiment.*

For any QPT $\mathcal{A}$, it holds that

$$\mathsf{Adv}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda,0)\to 1\right]-\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda,1)\to 1\right]\right| \le \mathsf{negl}(\lambda).$$

4.2 Definitions of SKE-CR-SKL

The syntax of SKE-CR-SKL is defined as follows.

Definition 4.4 (SKE-CR-SKL). *An SKE-CR-SKL scheme* $\mathsf{SKE\text{-}CR\text{-}SKL}$ *is a tuple of five algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{D}ec, \mathcal{V}rfy)$. *Below, let $\mathcal{M}$ be the message space of* $\mathsf{SKE\text{-}CR\text{-}SKL}$.

$\mathsf{Setup}(1^\lambda)\to \mathsf{msk}$**:** *The setup algorithm takes a security parameter 1^λ and outputs a master secret-key* msk.

$\mathcal{KG}(\mathsf{msk})\to(\mathit{dk},\mathsf{vk},\mathsf{tk})$**:** *The key generation algorithm takes the master secret-key* msk *as input. It outputs a decryption key dk, a verification key* vk, *and a testing key* tk.

$\mathsf{Enc}(\mathsf{msk},\mathsf{m})\to \mathsf{ct}$**:** *The encryption algorithm takes the master secret-key* msk *and a message $\mathsf{m}\in\mathcal{M}$, and outputs a ciphertext* ct.

$\mathcal{D}ec(\mathit{dk},\mathsf{ct})\to \widetilde{\mathsf{m}}$**:** *The decryption algorithm takes a decryption key dk and a ciphertext* ct, *and outputs a value* $\widetilde{\mathsf{m}}$.

$\mathcal{V}rfy(\mathsf{vk},\widetilde{\mathit{dk}})\to \top/\bot$**:** *The verification algorithm takes a verification key* vk *and a (possibly malformed) decryption key $\widetilde{\mathit{dk}}$, and outputs $\top$ or $\bot$.*

Decryption correctness: *For all $\mathsf{m}\in\mathcal{M}$, we have*

$$\Pr\left[\mathcal{D}ec(\mathit{dk},\mathsf{ct})=\mathsf{m} \;:\; \begin{array}{l}\mathsf{msk}\leftarrow\mathsf{Setup}(1^\lambda)\\ (\mathit{dk},\mathsf{vk},\mathsf{tk})\leftarrow\mathcal{KG}(\mathsf{msk})\\ \mathsf{ct}\leftarrow\mathsf{Enc}(\mathsf{msk},\mathsf{m})\end{array}\right]\ge 1-\mathsf{negl}(\lambda).$$

Verification correctness: *We have*

$$\Pr\left[\mathcal{V}rfy(\mathsf{vk},\mathit{dk})=\top \;:\; \begin{array}{l}\mathsf{msk}\leftarrow\mathsf{Setup}(1^\lambda)\\ (\mathit{dk},\mathsf{vk},\mathsf{tk})\leftarrow\mathcal{KG}(\mathsf{msk})\end{array}\right]\ge 1-\mathsf{negl}(\lambda).$$

Definition 4.5 (Classical Decryption Property). *We say that* $\mathsf{SKE\text{-}CR\text{-}SKL}=(\mathsf{Setup},\mathcal{KG},\mathsf{Enc},\mathcal{D}ec,\mathcal{V}rfy)$ *has the classical decryption property if there exists a deterministic polynomial time algorithm* CDec *such that given dk in the register* DK *and ciphertext* ct, *$\mathcal{D}ec$ applies the map $|u\rangle_{\mathsf{DK}}|v\rangle_{\mathsf{MSG}}\to|u\rangle_{\mathsf{DK}}|v\oplus\mathsf{CDec}(u,\mathsf{ct})\rangle_{\mathsf{MSG}}$ and outputs the measurement result of the register* MSG *in the computational basis, where* MSG *is initialized to $|0\cdots0\rangle_{\mathsf{MSG}}$.*

Definition 4.6 (Key Testability). *We say that an SKE-CR-SKL scheme* $\mathsf{SKE\text{-}CR\text{-}SKL}$ *with the classical decryption property satisfies key testability, if there exists a classical deterministic algorithm* $\mathsf{KeyTest}$ *that satisfies the following conditions:*

- ***Syntax:*** $\mathsf{KeyTest}$ *takes as input a testing key* tk *and a classical string* dk *as input. It outputs* 0 *or* 1.
- ***Correctness:*** *Let* $\mathsf{msk}\leftarrow\mathsf{Setup}(1^\lambda)$ *and* $(\mathit{dk},\mathsf{vk},\mathsf{tk})\leftarrow\mathcal{KG}(\mathsf{msk})$. *We denote the register holding dk as* DK. *Let* KT *be a register that is initialized to $|0\rangle_{\mathsf{KT}}$. If we apply the map $|u\rangle_{\mathsf{DK}}|\beta\rangle_{\mathsf{KT}}\to|u\rangle_{\mathsf{DK}}|\beta\oplus\mathsf{KeyTest}(\mathsf{tk},u)\rangle_{\mathsf{KT}}$ to the registers* DK *and* KT *and then measure* KT *in the computational basis, we obtain* 1 *with overwhelming probability.*
- ***Security:*** *Consider the following experiment* $\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda)$.

1. *The challenger $\mathcal{Ch}$ runs* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$ *and initializes $\mathcal{A}$ with input* msk.
2. *$\mathcal{A}$ requests q decryption keys for some polynomial q. $\mathcal{Ch}$ generates* $(d\mathcal{k}_i, \mathsf{vk}_i, \mathsf{tk}_i) \leftarrow \mathcal{KG}(\mathsf{msk})$ *for every* $i \in [q]$ *and sends* $(d\mathcal{k}_i, \mathsf{vk}_i, \mathsf{tk}_i)_{i\in[q]}$ *to $\mathcal{A}$.*
3. *$\mathcal{A}$ sends* $(k, \mathsf{dk}, \mathsf{m})$ *to $\mathcal{Ch}$, where k is an index,* dk *is a classical string and* m *is a message. $\mathcal{Ch}$ generates* $\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, \mathsf{m})$. *$\mathcal{Ch}$ outputs $\top$ if* $\mathsf{KeyTest}(\mathsf{tk}_k, \mathsf{dk}) = 1$ *and* $\mathsf{CDec}(\mathsf{dk}, \mathsf{ct}) \neq \mathsf{m}$. *Otherwise, $\mathcal{Ch}$ outputs* $\bot$.

For all QPT $\mathcal{A}$, the following must hold:

$$\mathsf{Adv}^{\mathsf{key\text{-}test}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \Pr\left[\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) \to \top\right] \leq \mathsf{negl}(\lambda).$$

Definition 4.7 (OT-IND-KLA Security). *We say that an SKE-CR-SKL scheme with key testability* SKE-CR-SKL *is OT-IND-KLA secure, if it satisfies the following requirement, formalized by the experiment* $\mathsf{Exp}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary $\mathcal{A}$ and a challenger $\mathcal{Ch}$:*

1. *$\mathcal{Ch}$ runs* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$ *and initializes $\mathcal{A}$ with the security parameter* 1^λ.
2. *$\mathcal{A}$ requests q decryption keys for some polynomial q. The challenger generates* $(d\mathcal{k}_i, \mathsf{vk}_i, \mathsf{tk}_i) \leftarrow \mathcal{KG}(\mathsf{msk})$ *for every* $i \in [q]$ *and sends* $(d\mathcal{k}_i, \mathsf{tk}_i)_{i\in[q]}$ *to $\mathcal{A}$.*
3. *$\mathcal{A}$ can get access to the following (stateful) verification oracle $O_{\mathcal{V}rfy}$ where V_i is initialized to be $\bot$:*

   $O_{\mathcal{V}rfy}(i, \widetilde{d\mathcal{k}})$**:** *It runs* $d \leftarrow \mathcal{V}rfy(\mathsf{vk}_i, \widetilde{d\mathcal{k}})$ *and returns d.*
   If $V_i = \bot$ and $d = \top$, it updates $V_i := \top$.
4. *$\mathcal{A}$ sends* $(\mathsf{m}^*_0, \mathsf{m}^*_1) \in \mathcal{M}^2$ *to the challenger. If $V_i = \bot$ for some $i \in [q]$, $\mathcal{Ch}$ outputs* 0 *as the final output of this experiment. Otherwise, $\mathcal{Ch}$ generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{msk}, \mathsf{m}^*_{\mathsf{coin}})$ *and sends* ct^* *to $\mathcal{A}$.*
5. *$\mathcal{A}$ outputs a guess* coin' *for* coin. *$\mathcal{Ch}$ outputs* coin' *as the final output of the experiment.*

For all QPT $\mathcal{A}$, it holds that:

$$\mathsf{Adv}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

5 SKE-CR-SKL with Key Testability

In this section, we show how to achieve SKE-CR-SKL introduced in Section 4.2.

5.1 Construction

We construct an SKE-CR-SKL scheme with key testability $\mathsf{SKE\text{-}CR\text{-}SKL} = \mathsf{SKE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{D}ec, \mathcal{V}rfy)$ having the additional algorithms CDec and KeyTest, using the following building blocks.

- BB84-based SKE-CD scheme (Definition 3.18) $\mathsf{SKECD} = \mathsf{SKECD}.(\mathsf{KG}, \mathcal{E}nc, \mathcal{D}ec, \mathcal{D}el, \mathsf{Vrfy})$ having the classical decryption algorithm SKECD.CDec.
- OWF $f : \{0,1\}^\lambda \to \{0,1\}^{p(\lambda)}$ for some polynomial p.

Let $\mathcal{M} := \{0,1\}^{\ell_\mathsf{m}}$ be the plaintext space. The construction is as follows:

$\mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$:

1. Generate $r \leftarrow \{0,1\}^{\ell_{\mathsf{m}}}$.
2. Generate $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$.
3. Output $\mathsf{msk} := (\mathsf{skecd.sk}, r)$.

$\mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk})$:

1. Parse $\mathsf{msk} = (\mathsf{skecd.sk}, r)$.
2. Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk}, r)$. $\mathsf{skecd.vk}$ is of the form $(x,\theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, and $\mathsf{skecd}.ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.
3. Generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $i \in [\ell_{\mathsf{ct}}]$ and $b \in \{0,1\}$. Set $T := t_{1,0}\|t_{1,1}\| \cdots \|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1}$.
4. Prepare a register $\mathsf{S_i}$ that is initialized to $\left|0^\lambda\right\rangle_{\mathsf{S_i}}$ for every $i \in [\ell_{\mathsf{ct}}]$.
5. For every $i \in [\ell_{\mathsf{ct}}]$, apply the map
$$|u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i\rangle_{\mathsf{S_i}} \to |u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S_i}}$$
to the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ and obtain the resulting state ρ_i.
6. Output $\mathit{dk} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$, $\mathsf{vk} = (x,\theta,S)$, and $\mathsf{tk} = T$.

$\mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{msk}, \mathsf{m})$:

1. Parse $\mathsf{msk} = (\mathsf{skecd.sk}, r)$.
2. Output $\mathsf{ct} := (\mathsf{skecd.sk}, r \oplus \mathsf{m})$.

$\mathsf{SKE\text{-}CR\text{-}SKL.CDec}(\mathsf{dk}, \mathsf{ct})$:

1. Parse $\mathsf{ct} = (\mathsf{skecd.sk}, z)$. Let $\widetilde{\mathsf{dk}}$ be the sub-string of dk on register $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$.
2. Output $z \oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{dk}})$.

$\mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{D}ec(\mathit{dk}, \mathsf{ct})$:

1. Parse $(\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$. We denote the register holding ρ_i as $\mathsf{SKECD.CT_i} \otimes \mathsf{S_i}$ for every $i \in [\ell_{\mathsf{ct}}]$.
2. Prepare a register MSG of ℓ_{m} qubits that is initialized to $|0\cdots0\rangle_{\mathsf{MSG}}$.
3. Apply the map
$$|u\rangle_{\bigotimes_{i\in[\ell_{\mathsf{ct}}]}\mathsf{SKECD.CT_i}} \otimes |w\rangle_{\mathsf{MSG}} \to$$
$$|u\rangle_{\bigotimes_{i\in[\ell_{\mathsf{ct}}]}\mathsf{SKECD.CT_i}} \otimes |w \oplus \mathsf{SKE\text{-}CR\text{-}SKL.CDec}(u, \mathsf{ct})\rangle_{\mathsf{MSG}}$$
to the registers $\bigotimes_{i\in[\ell_{\mathsf{ct}}]}\mathsf{SKECD.CT_i}$ and MSG.
4. Measure MSG in the computational basis and output the result m'.

$\mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, \widetilde{\mathit{dk}})$:

1. Parse $\mathsf{vk} = (x,\theta,S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1})$ and $\mathit{dk} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$ where ρ_i is a state on the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$.
2. For every $i \in [\ell_{\mathsf{ct}}]$, measure ρ_i in the Hadamard basis to get outcomes c_i, d_i corresponding to the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ respectively.

3. Output $\top$ if $x[i] = c_i \oplus d_i \cdot (s_{i,0} \oplus s_{i,1})$ holds for every $i \in [\ell_{\mathsf{ct}}]$ such that $\theta[i] = 1$. Otherwise, output $\bot$.

$\mathsf{SKE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{tk}, \mathsf{dk})$:

1. Parse dk as a string over the registers $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$. Let u_i denote the value on $\mathsf{SKECD.CT_i}$ and v_i the value on $\mathsf{S_i}$. Parse tk as $T = t_{1,0} \| t_{1,1} \| \cdots \| t_{\ell_{\mathsf{ct}},0} \| t_{\ell_{\mathsf{ct}},1}$.
2. Let $\mathsf{Check}[t_{i,0}, t_{i_1}](u_i, v_i)$ be the deterministic algorithm that outputs 1 if $f(v_i) = t_{i,u_i}$ holds and 0 otherwise.
3. Output $\mathsf{Check}[t_{1,0}, t_{1,1}](u_1, v_1) \wedge \cdots \wedge \mathsf{Check}[t_{\ell_{\mathsf{ct}},0}, t_{\ell_{\mathsf{ct}},1}](u_{\ell_{\mathsf{ct}}}, v_{\ell_{\mathsf{ct}}})$.

Decryption correctness: For a ciphertext $\mathsf{ct} = (\mathsf{skecd.sk}, r \oplus \mathsf{m})$, the decryption algorithm $\mathcal{D}ec$ performs the following computation:

$$|u\rangle_{\bigotimes_{i\in[\ell_{\mathsf{ct}}]} \mathsf{SKECD.CT_i}} \otimes |w\rangle_{\mathsf{MSG}} \rightarrow$$
$$|u\rangle_{\bigotimes_{i\in[\ell_{\mathsf{ct}}]} \mathsf{SKECD.CT_i}} \otimes |w \oplus r \oplus \mathsf{m} \oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u)\rangle_{\mathsf{MSG}}$$

Recall that $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ is a register corresponding to the ciphertext of a BB84-based SKE-CD scheme SKECD. Hence, from the Classical Decryption property of SKECD (Definition 3.18), it must be that $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u) = r$ for every u in the superposition of $\mathsf{skecd}.ct$. Consequently, m is written onto the MSG register in each term of the superposition and decryption correctness follows.

Verification correctness: Observe that for the Hadamard basis positions ($i \in [\ell_{\mathsf{ct}}]$ such that $\theta[i] = 1$), ρ_i is of the form:

$$\rho[i] = |0\rangle_{\mathsf{SKECD.CT_i}} |s_{i,0}\rangle_{\mathsf{S_i}} + (-1)^{x[i]} |1\rangle_{\mathsf{SKECD.CT_i}} |s_{i,1}\rangle_{\mathsf{S_i}}$$

It is easy to see that measuring the state in the Hadamard basis gives outcomes c_i, d_i (on registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ respectively) satisfying $x[i] = c_i \oplus d_i \cdot (s_{i,0} \oplus s_{i,1})$. Hence, the verification correctness follows.

Theorem 5.1. *There exists an SKE-CR-SKL scheme satisfying OT-IND-KLA security and Key-Testability, assuming the existence of a BB84-based SKE-CD scheme and the existence of an OWF.*

We prove this theorem in the subsequent sections.

5.2 Proof of OT-IND-KLA Security

Let $\mathcal{A}$ be an adversary for the OT-IND-KLA security of the construction $\mathsf{SKE\text{-}CR\text{-}SKL}$ that makes use of a BB84-based SKE-CD scheme SKECD. Consider the hybrid $\mathsf{Hyb}_j^{\mathsf{coin}}$ defined as follows:

$\mathsf{Hyb}_j^{\mathsf{coin}}$:

1. The challenger $\mathcal{C}h$ runs $\mathsf{msk} \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{A}$ with input 1^λ.
2. $\mathcal{A}$ requests q decryption keys for some polynomial q. For each $k \in [j]$, $\mathcal{C}h$ generates $(\mathit{dk}_i, \mathsf{vk}_i, \mathsf{tk}_i) \leftarrow \widetilde{\mathcal{KG}}(\mathsf{msk})$ where $\widetilde{\mathcal{KG}}$ is defined as follows (the difference from $\mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}$ is colored in red):

$\widetilde{\mathcal{KG}}(\mathsf{msk})$:

(a) Parse $\mathsf{msk} = (\mathsf{skecd.sk}, r)$.
(b) Sample $\widetilde{r} \leftarrow \{0,1\}^{\ell_{\mathsf{m}}}$.
(c) Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk}, \widetilde{r})$. $\mathsf{skecd.vk}$ is of the form $(x, \theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, and $\mathsf{skecd}.ct$ can be described as $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.

(d) Generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $i \in [\ell_{\mathsf{ct}}]$ and $b \in \{0,1\}$. Set $T := t_{1,0}\|t_{1,1}\|\cdots\|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1}$.

(e) Prepare a register $\mathsf{S_i}$ that is initialized to $|0\cdots 0\rangle_{\mathsf{S_i}}$ for every $i \in [\ell_{\mathsf{ct}}]$.

(f) For every $i \in [\ell_{\mathsf{ct}}]$, apply the map

$$|u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i\rangle_{\mathsf{S_i}} \to |u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S_1}}$$

to the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ and obtain the resulting state ρ_i.

(g) Output $\widetilde{dk} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$, $\mathsf{vk} = (x,\theta,S)$, and $\mathsf{tk} = T$.

3. On the other hand, for $k = j+1,\ldots,q$, $\mathcal{Ch}$ generates $(\widetilde{dk}_k, \mathsf{vk}_k, \mathsf{tk}_k) \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk})$. Then, $\mathcal{Ch}$ sends $(\widetilde{dk}_k, \mathsf{tk}_k)_{k\in[q]}$ to $\mathcal{A}$.
4. $\mathcal{A}$ can get access to the following (stateful) verification oracle $O_{\mathcal{V}rfy}$ where V_i is initialized to $\bot$:

   $O_{\mathcal{V}rfy}(i, \widetilde{dk})$**:** It runs $d \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}_i, \widetilde{dk})$ and returns d. If $V_i = \bot$ and $d = \top$, it updates $V_i := \top$.
5. $\mathcal{A}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \{0,1\}^{\ell_{\mathsf{m}}} \times \{0,1\}^{\ell_{\mathsf{m}}}$ to $\mathcal{Ch}$. If $V_i = \bot$ for some $i \in [q]$, $\mathcal{Ch}$ outputs 0 as the final output of this experiment. Otherwise, $\mathcal{Ch}$ generates $\mathsf{ct}^* \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{msk}, \mathsf{m}^*_{\mathsf{coin}})$ and sends ct^* to $\mathcal{A}$.
6. $\mathcal{A}$ outputs a guess coin' for coin. The challenger outputs coin' as the final output of the experiment.

We will now prove the following lemma:

Lemma 5.2. $\forall j \in \{0,\ldots,q-1\}$ *and* $\mathsf{coin} \in \{0,1\} : \mathsf{Hyb}_j^{\mathsf{coin}} \approx \mathsf{Hyb}_{j+1}^{\mathsf{coin}}$.

Proof. Suppose $\mathsf{Hyb}_j^{\mathsf{coin}} \not\approx \mathsf{Hyb}_{j+1}^{\mathsf{coin}}$. Let $\mathcal{D}$ be a corresponding distinguisher. We will construct a reduction $\mathcal{R}$ that breaks the IND-CVA-CD security of the BB84-based SKE-CD scheme SKECD. The execution of $\mathcal{R}^{\mathcal{D}}$ in the experiment $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{R}}(1^\lambda, b)$ proceeds as follows:

Execution of $\mathcal{R}^{\mathcal{D}}$ in $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{R}}(1^\lambda, b)$:

1. The challenger $\mathcal{Ch}$ computes $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$.
2. $\mathcal{R}$ samples $(r_0, r_1) \leftarrow \{0,1\}^{\ell_{\mathsf{m}}} \times \{0,1\}^{\ell_{\mathsf{m}}}$ and sends it to $\mathcal{Ch}$.
3. $\mathcal{Ch}$ computes $(\mathsf{skecd}.ct^\star, \mathsf{skecd.vk}^\star) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk}, r_b)$ and sends $\mathsf{skecd}.ct^\star$ to $\mathcal{R}$.
4. $\mathcal{R}$ initializes $\mathcal{D}$ with 1^λ. $\mathcal{D}$ requests q keys for some polynomial q.
5. For each $k \in [j]$, $\mathcal{R}$ computes $\widetilde{dk}_k$ as follows:
   - Sample a random value $\widetilde{r} \leftarrow \{0,1\}^{\ell_{\mathsf{m}}}$.
   - Compute $(\mathsf{skecd}.\widetilde{ct}, \mathsf{skecd}.\widetilde{\mathsf{vk}}) \leftarrow O_{\mathcal{E}nc}(\widetilde{r})$.
   - Compute $\widetilde{dk}_k$ by executing Steps 2.(c)-2.(g) as in Hyb_j^0, but using $\mathsf{skecd}.\widetilde{ct}$ in place of $\mathsf{skecd}.ct$.
6. $\mathcal{R}$ computes $\widetilde{dk}_{j+1}$ by executing Steps 2.(c)-2.(g) as in Hyb_j^0, but using $\mathsf{skecd}.ct^\star$ in place of $\mathsf{skecd}.ct$.
7. For each $k \in [j+2, q]$, $\mathcal{R}$ computes $\widetilde{dk}_k$ as follows:
   - Compute $(\mathsf{skecd}.\widetilde{ct}, \mathsf{skecd}.\widetilde{\mathsf{vk}}) \leftarrow O_{\mathcal{E}nc}(r_1)$.
   - Compute $\widetilde{dk}_k$ by executing Steps 2.(c)-2.(g) as in Hyb_j^0, but using $\mathsf{skecd}.\widetilde{ct}$ in place of $\mathsf{skecd}.ct$.
8. $\mathcal{R}$ sends $\widetilde{dk}_1, \ldots, \widetilde{dk}_q$ to $\mathcal{D}$ and initializes $V_k = \bot$ for every $k \in [q]$.
9. If $k \neq j+1$, $\mathcal{R}$ simulates the response of oracle $O_{\mathcal{V}rfy}(k, \widetilde{dk})$ as follows:
   - Parse $\widetilde{\mathsf{vk}} = (x, \theta, S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1})$ and $\widetilde{dk} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$.
   - For every $i \in [\ell_{\mathsf{ct}}]$, measure ρ_i in the Hadamard basis to get outcomes c_i, d_i corresponding to the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ respectively.

- Compute $\mathsf{cert}[i] = c_i \oplus d_i \cdot (s_{i,0} \oplus s_{i,1})$ for every $i \in [\ell_{\mathsf{ct}}]$.
- If $x[i] = \mathsf{cert}[i]$ holds for every $i \in [\ell_{\mathsf{ct}}] : \theta[i] = 1$, then update $V_k = \top$ and send $\top$ to $\mathcal{D}$. Else, send $\bot$.

10. If $k = j+1$, $\mathcal{R}$ simulates the response of oracle $O_{\mathcal{V}\mathit{rfy}}(k, \widetilde{d\mathcal{K}})$ as follows:
    - Compute $\mathsf{cert} = \mathsf{cert}[1] \| \ldots \| \mathsf{cert}[\ell_{\mathsf{ct}}]$, where each $\mathsf{cert}[i]$ is computed as in Step 9.
    - Send cert to $\mathcal{C}\mathit{h}$. If $\mathcal{C}\mathit{h}$ returns skecd.sk, send $\top$ to $\mathcal{D}$ and update $V_{j+1} = \top$. Else if $\mathcal{C}\mathit{h}$ returns $\bot$, send $\bot$ to $\mathcal{D}$.
11. $\mathcal{D}$ sends $(\mathsf{m}_0^\star, \mathsf{m}_1^\star) \in \{0,1\}^{\ell_{\mathsf{m}}} \times \{0,1\}^{\ell_{\mathsf{m}}}$ to $\mathcal{R}$. If $V_i = \bot$ for any $i \in [q]$, $\mathcal{R}$ sends 0 to $\mathcal{C}\mathit{h}$. $\mathcal{R}$ computes $\mathsf{ct}^\star = (\mathsf{skecd.sk}, r_1 \oplus \mathsf{m}^\star_{\mathsf{coin}})$, where skecd.sk is obtained from $\mathcal{C}\mathit{h}$ in Step 10. $\mathcal{R}$ sends ct^* to $\mathcal{D}$.
12. $\mathcal{D}$ outputs a guess b' which $\mathcal{R}$ forwards to $\mathcal{C}\mathit{h}$. $\mathcal{C}\mathit{h}$ outputs b' as the final output of the experiment.

We will first argue that when $b = 1$, the view of $\mathcal{D}$ is exactly the same as its view in the hybrid $\mathsf{Hyb}^{\mathsf{coin}}_j$. Notice that the reduction computes the first j decryption keys by querying the encryption oracle on random plaintexts. Hyb_j on the other hand, directly computes them but there is no difference in the output ciphertexts. A similar argument holds for the keys $d\mathcal{K}_{j+2}, \ldots, d\mathcal{K}_q$, which contain encryptions of the same random value r_1. Moreover, if $b = 1$, the value encrypted as part of the key $d\mathcal{K}_{j+1}$ is also r_1. This is the same as in $\mathsf{Hyb}^{\mathsf{coin}}_j$. As for the verification oracle queries, notice that they are answered similarly by the reduction and $\mathsf{Hyb}^{\mathsf{coin}}_j$ for all but the $j+1$-th key. For the $j+1$-th key, the reduction works differently in that it forwards the certificate cert to the verification oracle. However, the verification procedure of the BB84-based SKE-CD scheme checks the validity of the value cert in the same way as the reduction, so there is no difference.

Finally, notice that when $b = 0$, the encrypted value is random and independent of r_1, similar to the hybrid $\mathsf{Hyb}^{\mathsf{coin}}_{j+1}$. Consequently, $\mathcal{R}$ breaks the IND-CVA-CD security of SKECD with non-negligible probability, a contradiction. □

Notice that hybrid $\mathsf{Hyb}^{\mathsf{coin}}_0$ is the same as experiment $\mathsf{Exp}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$. From the previous lemma, we have that $\mathsf{Hyb}^{\mathsf{coin}}_0 \approx \mathsf{Hyb}^{\mathsf{coin}}_q$. However, we have that $\mathsf{Hyb}^0_q \approx \mathsf{Hyb}^1_q$ because Hyb^0_q and Hyb^1_q do not encrypt r at all as part of the decryption keys, but they mask the plaintext with r. Consequently, we have that $\mathsf{Hyb}^0_0 \approx \mathsf{Hyb}^1_0$, which completes the proof. □

5.3 Proof of Key-Testability

First, we will argue the correctness requirement. Recall that $\mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}$ applies the following map to a BB84 state $|x\rangle_\theta$ where $(x,\theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$ for every $i \in [\ell_{\mathsf{ct}}]$:

$$|u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i\rangle_{\mathsf{S_i}} \to |u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i \oplus s_{i,u_i}\rangle_{\mathsf{S_i}}$$

where $\mathsf{SKECD.CT_i}$ denotes the register holding the i-th qubit of $|x\rangle_\theta$ and $\mathsf{S_i}$ is a register initialized to $|0\ldots0\rangle_{\mathsf{S_i}}$.

Consider applying the algorithm SKE-CR-SKL.KeyTest in superposition to the resulting state, i.e., performing the following map, where $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$, and KT is initialized to $|0\rangle$:

$$|u\rangle_{\mathsf{SKECD.CT}} |v\rangle_{\mathsf{S}} |\beta\rangle_{\mathsf{KT}} \to |u\rangle_{\mathsf{SKECD.CT}} |v\rangle_{\mathsf{S}} |\beta \oplus \mathsf{SKE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{tk}, u\|v)\rangle_{\mathsf{KT}}$$

where $\mathsf{tk} = T = t_{1,0}\|t_{1,1}\| \cdots \|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$. Recall that SKE-CR-SKL.KeyTest outputs 1 if and only if $\mathsf{Check}[t_{i,0}, t_{i,1}](u_i, v_i) = 1$ for every $i \in [\ell_{\mathsf{ct}}]$, where u_i, v_i denote the states of the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ respectively. Recall that $\mathsf{Check}[t_{i,0}, t_{i,1}](u_i, v_i)$ computes $f(v_i)$ and checks if it equals t_{i,u_i}. Since the construction chooses t_{i,u_i} such that $f(s_{i,u_i}) = t_{i,u_i}$, this check always passes. Consequently, measuring register KT always produces outcome 1.

We will now argue that the security requirement holds by showing the following reduction to the security of the OWF f. Let $\mathcal{A}$ be an adversary that breaks the key-testability of SKE-CR-SKL. Consider a QPT reduction $\mathcal{R}$ that works as follows in the OWF security experiment:

Execution of $\mathcal{R}^{\mathcal{A}}$ in $\mathsf{Expt}^{\mathsf{owf}}_{f,\mathcal{R}}(1^\lambda)$:

1. The challenger chooses $s \leftarrow \{0,1\}^\lambda$ and sends $y := f(s)$ to $\mathcal{R}$.
2. $\mathcal{R}$ runs $\mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{A}$ with input msk.
3. $\mathcal{A}$ requests q decryption keys for some polynomial q. $\mathcal{R}$ picks a random index $k^\star \in [q]$. For every $k \neq k^\star$, $\mathcal{R}$ generates $(d\mathcal{k}_k, \mathsf{vk}_k, \mathsf{tk}_k)$ by computing the function f as needed. For the index $k^\star$, $\mathcal{R}$ computes $(d\mathcal{k}_{k^\star}, \mathsf{vk}_{k^\star}, \mathsf{tk}_{k^\star})$ as follows (the difference is colored in red):
   (a) Parse $\mathsf{msk} = (\mathsf{skecd.sk}, r)$.
   (b) Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk}, r)$. $\mathsf{skecd.vk}$ is of the form $(x,\theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, $\mathsf{skecd}.ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.
   (c) Choose an index $i^\star \in [\ell_{\mathsf{ct}}]$ such that $\theta[i^\star] = 0$. For every $i \in [\ell_{\mathsf{ct}}]$ such that $i \neq i^\star$, generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $b \in \{0,1\}$. For $i = i^\star$, set $t_{i^\star,1-x[i^\star]} := y$. Then, generate $s_{i^\star,x[i^\star]} \leftarrow \{0,1\}^\lambda$ and compute $t_{i^\star,x[i^\star]} = f(s_{i^\star,x[i^\star]})$. Set $T := t_{1,0} \| t_{1,1} \| \cdots \| t_{\ell_{\mathsf{ct}},0} \| t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i \in [\ell_{\mathsf{ct}}] : \theta[i]=1}$.
   (d) Prepare a register S_i that is initialized to $|0\cdots0\rangle_{\mathsf{S}_\mathsf{i}}$ for every $i \in [\ell_{\mathsf{ct}}]$.
   (e) For every $i \in [\ell_{\mathsf{ct}}]$, apply the map
   $$|u_i\rangle_{\mathsf{SKECD.CT}_\mathsf{i}} \otimes |v_i\rangle_{\mathsf{S}_\mathsf{i}} \to |u_i\rangle_{\mathsf{SKECD.CT}_\mathsf{i}} \otimes |v_i \oplus s_{i,u_i}\rangle_{\mathsf{S}_\mathsf{i}}$$
   to the registers $\mathsf{SKECD.CT}_\mathsf{i}$ and S_i and obtain the resulting state ρ_i.
   (f) Compute $d\mathcal{k}_{k^\star} = (\rho_i)_{i \in [\ell_{\mathsf{ct}}]}$, $\mathsf{vk}_{k^\star} = (x, \theta, S)$, and $\mathsf{tk}_{k^\star} = T$.
4. $\mathcal{R}$ sends $(d\mathcal{k}_i, \mathsf{vk}_i, \mathsf{tk}_i)$ to $\mathcal{A}$ for every $i \in [q]$.
5. $\mathcal{A}$ sends $(k, \mathsf{dk}, \mathsf{m})$ to $\mathcal{R}$. If $k \neq k^\star$, $\mathcal{R}$ aborts. Otherwise, $\mathcal{R}$ parses dk as a string over the registers $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$ and measures the register $\mathsf{S}_{\mathsf{i}^\star}$ to obtain an outcome $s_{i^\star}$. $\mathcal{R}$ then sends $s_{i^\star}$ to the challenger.

Notice that the view of $\mathcal{A}$ is the same as its view in the key-testability experiment, as only the value $t_{i^\star,1-x[i^\star]}$ is generated differently by forwarding the value y, but this value is distributed identically to the original value. Note that in both cases, $\mathcal{A}$ receives no information about a pre-image of $t_{i^\star,1-x[i^\star]}$. Now, $\mathcal{R}$ guesses the index k that $\mathcal{A}$ targets with probability $\frac{1}{q}$. By assumption, we have that $\mathsf{CDec}(\mathsf{dk}, \mathsf{ct}) \neq \mathsf{m}$ where $\mathsf{ct} = \mathsf{Enc}(\mathsf{msk}, \mathsf{m})$. The value dk can be parsed as a string over the registers $\mathsf{SKECD.CT}$ and S. Let $\widetilde{\mathsf{dk}}$ be the sub-string of dk on the register $\mathsf{SKECD.CT}$. Recall that CDec invokes the algorithm $\mathsf{SKECD.CDec}$ on input $\widetilde{\mathsf{dk}}$. We will now recall a property of $\mathsf{SKECD.CDec}$ that was specified in Definition 3.18:

Let $(ct, \mathsf{vk} = (x,\theta)) \leftarrow \mathsf{SKECD.Enc}(\mathsf{skecd.sk}, r)$ where $\mathsf{skecd.sk}$ is generated as $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$. Now, let u be any arbitrary value such that $u[i] = x[i]$ for all $i : \theta[i] = 0$. Then, the following holds:

$$\Pr\left[\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u) = r\right] \geq 1 - \mathsf{negl}(\lambda)$$

Consequently, if $\widetilde{\mathsf{dk}}$ is such that $\widetilde{\mathsf{dk}}[i] = x[i]$ for all $i : \theta[i] = 0$, where (x,θ) are specified by $\mathsf{vk}_{k^\star}$, then $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{dk}})$ outputs the value r with high probability. Since $\mathsf{CDec}(\mathsf{dk}, \mathsf{ct} = (\mathsf{skecd.sk}, r \oplus \mathsf{m}))$ outputs $r \oplus \mathsf{m} \oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{dk}})$, we have that $\mathsf{CDec}(\mathsf{dk}, \mathsf{ct} = (\mathsf{skecd.sk}, r \oplus \mathsf{m})) = \mathsf{m}$. Therefore, it must be the case that there exists some index i for which $\widetilde{\mathsf{dk}}[i] \neq x[i]$. With probability $\frac{1}{\ell_{\mathsf{ct}}}$, this happens to be the guessed value $i^\star$. In this case, $\mathcal{A}$ must output $s_{i^\star}$ on register S_i such that $f(s_{i^\star}) = t_{i^\star,1-x[i^\star]} = y$. This concludes the proof. □

Since we have proved OT-IND-KLA security (Section 5.2) and Key-Testability (Section 5.3), we can now state the following theorem:

6 PKE-CR-SKL from LWE

In this section, we show how to achieve PKE-CR-SKL from SKE-CR-SKL, standard ABE, and compute-and-compare obfuscation.

6.1 Construction

We construct a PKE-CR-SKL scheme $\mathsf{PKE\text{-}CR\text{-}SKL} = \mathsf{PKE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$ with message space $\mathcal{M} = \{0,1\}^\ell$ using the following building blocks.

- ABE scheme $\mathsf{ABE} = \mathsf{ABE}.(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$ for the following relation R.

  $R(x,y)$**:** Interpret x as a circuit. Then, output 0 (decryptable) if $\bot = x(y)$ and otherwise 1.

- Compute-and-Compare Obfuscation $\mathsf{CC.Obf}$ with the simulator $\mathsf{CC.Sim}$.
- SKE-CR-SKL scheme with Key Testability $\mathsf{SKE\text{-}CR\text{-}SKL} = \mathsf{SKE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy}, \mathsf{KeyTest})$. It also has the classical decryption algorithm $\mathsf{SKE\text{-}CR\text{-}SKL.CDec}$.

The construction is as follows.

$\mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$**:**

- Generate $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$.
- Generate $\mathsf{ske.msk} \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$.
- Output $\mathsf{ek} := \mathsf{abe.pk}$ and $\mathsf{msk} := (\mathsf{abe.msk}, \mathsf{ske.msk})$.

$\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk})$**:**

- Parse $\mathsf{msk} = (\mathsf{abe.msk}, \mathsf{ske.msk})$.
- Generate $(\mathsf{ske}.\mathcal{dk}, \mathsf{ske.vk}, \mathsf{ske.tk}) \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{ske.msk})$. We denote the register holding $\mathsf{ske}.\mathcal{dk}$ as $\mathsf{SKE.DK}$.
- Prepare a register $\mathsf{ABE.SK}$ that is initialized to $|0\cdots 0\rangle_{\mathsf{ABE.SK}}$.
- Choose explicit randomness $\mathsf{k} \leftarrow \{0,1\}^\lambda$.
- To the registers $\mathsf{SKE.DK}$ and $\mathsf{ABE.SK}$, apply the map

$$|u\rangle_{\mathsf{SKE.DK}} |v\rangle_{\mathsf{ABE.SK}} \to |u\rangle_{\mathsf{SKE.DK}} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k})\rangle_{\mathsf{ABE.SK}}$$

  and obtain $\mathcal{dk}$ over the registers $\mathsf{SKE.DK}$ and $\mathsf{ABE.SK}$.
- Output $\mathcal{dk}$ and $\mathsf{vk} := (\mathsf{abe.msk}, \mathsf{ske.vk}, \mathsf{ske.tk}, \mathsf{k})$.

$\mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ek}, \mathsf{m})$**:**

- Parse $\mathsf{ek} = \mathsf{abe.pk}$.
- Generate $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$, where pp_D consists of circuit parameters of D defined in the security proof.
- Generate $\mathsf{abe.ct} \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, \widetilde{C}, \mathsf{m})$.
- Output $\mathsf{ct} := \mathsf{abe.ct}$.

$\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{Dec}(\mathcal{dk}, \mathsf{ct})$**:**

- Parse $\mathsf{ct} = \mathsf{abe.ct}$. We denote the register holding $\mathcal{dk}$ as $\mathsf{SKE.DK} \otimes \mathsf{ABE.SK}$.
- Prepare a register MSG that is initialized to $|0\cdots 0\rangle_{\mathsf{MSG}}$

- Apply the map $|v\rangle_{\mathsf{ABE.SK}}|w\rangle_{\mathsf{MSG}} \to |v\rangle_{\mathsf{ABE.SK}}|w \oplus \mathsf{ABE.Dec}(v, \mathsf{abe.ct})\rangle_{\mathsf{MSG}}$ to the registers ABE.SK and MSG.
- Measure the register MSG in the computational basis and output the result m'.

$\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, d\mathcal{K}')$:

- Parse $\mathsf{vk} = (\mathsf{abe.msk}, \mathsf{ske.vk}, \mathsf{ske.tk}, \mathsf{k})$. We denote the register holding $d\mathcal{K}'$ as $\mathsf{SKE.DK} \otimes \mathsf{ABE.SK}$.
- Prepare a register SKE.KT that is initialized to $|0\rangle_{\mathsf{SKE.KT}}$.
- To the registers SKE.DK and SKE.KT, apply the map

$$|u\rangle_{\mathsf{SKE.DK}}|\beta\rangle_{\mathsf{SKE.KT}} \to |u\rangle_{\mathsf{SKE.DK}}|\beta \oplus \mathsf{SKE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{ske.tk}, u)\rangle_{\mathsf{SKE.KT}}.$$

- Measure SKE.KT in the computational basis and output $\bot$ if the result is 0. Otherwise, go to the next step.
- To the registers SKE.DK and ABE.SK, apply the map

$$|u\rangle_{\mathsf{SKE.DK}}|v\rangle_{\mathsf{ABE.SK}} \to |u\rangle_{\mathsf{SKE.DK}}|v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k})\rangle_{\mathsf{ABE.SK}}.$$

- Trace out the register ABE.SK and obtain $\mathsf{ske}.d\mathcal{K}'$ over SKE.DK.
- Output $\top$ if $\top = \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{ske.vk}, \mathsf{ske}.d\mathcal{K}')$ and $\bot$ otherwise.

Decryption correctness. The decryption key $d\mathcal{K}$ output by $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{K}\mathcal{G}$ is of the form $\sum_u \alpha_u |u\rangle_{\mathsf{SKE.DK}}|\mathsf{abe.sk}_u\rangle_{\mathsf{ABE.SK}}$, where $\mathsf{abe.sk}_u \leftarrow \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k})$. Let $\mathsf{ct} \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ek}, \mathsf{m})$. If we apply the map $|v\rangle_{\mathsf{ABE.SK}}|w\rangle_{\mathsf{MSG}} \to |v\rangle_{\mathsf{ABE.SK}}|w \oplus \mathsf{ABE.Dec}(v, \mathsf{abe.ct})\rangle_{\mathsf{MSG}}$ to $\sum_u \alpha_u |u\rangle_{\mathsf{SKE.DK}}|\mathsf{abe.sk}_u\rangle_{\mathsf{ABE.SK}}\left|0^\ell\right\rangle_{\mathsf{MSG}}$, with overwhelming probability, the result is negligibly close to

$$\sum_u \alpha_u |u\rangle_{\mathsf{SKE.DK}}|\mathsf{abe.sk}_u\rangle_{\mathsf{ABE.SK}} \otimes |\mathsf{m}\rangle_{\mathsf{MSG}}$$

since $\widetilde{C}(u) = \bot$ and thus $R(\widetilde{C}, u) = 0$ for $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$ and almost every string u. Therefore, we see that PKE-CR-SKL satisfies decryption correctness.

Verification correctness. Let $d\mathcal{K} \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{K}\mathcal{G}(\mathsf{msk})$. It is clear that the state $\mathsf{ske}.d\mathcal{K}'$ obtained when computing $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, d\mathcal{K})$ is the same as $\mathsf{ske}.d\mathcal{K}$ generated when generating $d\mathcal{K}$. Therefore, the verification correctness of PKE-CR-SKL follows from that of SKE-CR-SKL.

6.2 Proof of IND-KLA Security

Let $\mathcal{A}$ be an adversary for the IND-KLA security of PKE-CR-SKL. We consider the following sequence of experiments.

$\mathsf{Hyb}_0^{\mathsf{coin}}$: This is $\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$.

1. The challenger $\mathcal{C}h$ generates $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and $\mathsf{ske.msk} \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$, and sends $\mathsf{ek} := \mathsf{abe.pk}$ to $\mathcal{A}$.
2. $\mathcal{A}$ requests q decryption keys for some polynomial q. $\mathcal{C}h$ generates $d\mathcal{K}_i$ as follows for every $i \in [q]$:
   - Generate $(\mathsf{ske}.d\mathcal{K}_i, \mathsf{ske.vk}_i, \mathsf{ske.tk}_i) \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{K}\mathcal{G}(\mathsf{ske.msk})$. We denote the register holding $\mathsf{ske}.d\mathcal{K}_i$ as $\mathsf{SKE.DK_i}$.
   - Prepare a register $\mathsf{ABE.SK_i}$ that is initialized to $|0\cdots0\rangle_{\mathsf{ABE.SK_i}}$.
   - Choose explicit randomness $\mathsf{k}_i \leftarrow \{0,1\}^\lambda$.

- To $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.SK_i}$, apply the map

$$|u\rangle_{\mathsf{SKE.DK_i}}|v\rangle_{\mathsf{ABE.SK_i}} \to |u\rangle_{\mathsf{SKE.DK_i}}|v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k}_i)\rangle_{\mathsf{ABE.SK_i}}$$

and obtain $\widetilde{dk}_i$ over the registers $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.SK_i}$.

$\mathcal{CH}$ sends $\widetilde{dk}_1, \ldots, \widetilde{dk}_q$ to $\mathcal{A}$.

3. $\mathcal{A}$ can get access to the following (stateful) verification oracle $O_{\mathcal{V}rfy}$ where V_i is initialized to be $\bot$:

$O_{\mathcal{V}rfy}(i, \widetilde{dk})$**:** It computes d as follows.

(a) Let the register holding $\widetilde{dk}$ be $\mathsf{SKE.DK_i} \otimes \mathsf{ABE.SK_i}$.
(b) Prepare a register $\mathsf{SKE.KT_i}$ that is initialized to $|0\rangle_{\mathsf{SKE.KT_i}}$.
(c) To $\mathsf{SKE.DK_i}$ and $\mathsf{SKE.KT_i}$ apply the map:

$$|u\rangle_{\mathsf{SKE.DK_i}}|\beta\rangle_{\mathsf{SKE.KT_i}} \to |u\rangle_{\mathsf{SKE.DK_i}}|\beta \oplus \mathsf{SKE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{ske.tk}_i, u)\rangle_{\mathsf{SKE.KT_i}}.$$

(d) Measure $\mathsf{SKE.KT_i}$ in the computational basis and set $d := \bot$ if the result is 0. Otherwise, go to the next step.
(e) To $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.SK_i}$, apply the map

$$|u\rangle_{\mathsf{SKE.DK_i}}|v\rangle_{\mathsf{ABE.SK_i}} \to |u\rangle_{\mathsf{SKE.DK_i}}|v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k}_i)\rangle_{\mathsf{ABE.SK_i}}.$$

(f) Trace out register $\mathsf{ABE.SK_i}$ and obtain $\mathsf{ske}.dk'$ over $\mathsf{SKE.DK_i}$.
(g) Set $d := \top$ if $\top = \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{ske.vk}_i, \mathsf{ske}.dk')$ and set $d := \bot$ otherwise. It returns d to $\mathcal{A}$. If $V_i = \bot$ and $d = \top$, it updates $V_i := \top$.

4. $\mathcal{A}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{CH}$. If $V_i = \bot$ for some $i \in [q]$, $\mathcal{CH}$ outputs 0 as the final output of this experiment. Otherwise, $\mathcal{CH}$ generates $\widetilde{C}^* \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$ and $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, \widetilde{C}^*, \mathsf{m}^*_{\mathsf{coin}})$, and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{A}$.
5. $\mathcal{A}$ outputs coin'. $\mathcal{CH}$ outputs coin' as the final output of the experiment.

$\mathsf{Hyb}_1^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_0^{\mathsf{coin}}$ except that $\widetilde{C}^*$ is generated as $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{ske.ct}^*], \mathsf{lock}, 0)$, where $\mathsf{ske.ct}^* \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ske.msk}, 0^\lambda)$, $\mathsf{lock} \leftarrow \{0,1\}^\lambda$, and $D[\mathsf{ske.ct}^*](x)$ is a circuit that has $\mathsf{ske.ct}^*$ hardwired and outputs $\mathsf{SKE\text{-}CR\text{-}SKL.CDec}(x, \mathsf{ske.ct}^*)$.

We pick lock as a uniformly random string that is completely independent of other variables such as $\mathsf{ske.ct}^*$. Thus, from the security of CC.Obf, we have $\mathsf{Hyb}_0^{\mathsf{coin}} \approx \mathsf{Hyb}_1^{\mathsf{coin}}$.

$\mathsf{Hyb}_2^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_1^{\mathsf{coin}}$ except that $\mathsf{ske.ct}^*$ hardwired into the obfuscated circuit $\widetilde{C}^*$ is generated as $\mathsf{ske.ct}^* \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ske.msk}, \mathsf{lock})$.

From the OT-IND-KLA security of SKE-CR-SKL, we can show that $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$. Suppose that $\mathsf{Hyb}_1^{\mathsf{coin}} \not\approx \mathsf{Hyb}_2^{\mathsf{coin}}$ and $\mathcal{D}$ is a corresponding distinguisher. We consider the following reduction $\mathcal{R}$:

Execution of $\mathcal{R}^{\mathcal{D}}$ in $\mathsf{Exp}^{\mathsf{ot\text{-}ind\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{R}}(1^\lambda, b)$:

1. $\mathcal{CH}$ runs $\mathsf{ske.msk} \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{R}$ with the security parameter 1^λ.
2. $\mathcal{R}$ generates $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and sends $\mathsf{ek} := \mathsf{abe.pk}$ to $\mathcal{D}$.
3. $\mathcal{D}$ requests q decryption keys for some polynomial q. $\mathcal{R}$ requests q decryption keys. $\mathcal{CH}$ generates $(\mathsf{ske}.dk_i, \mathsf{ske.vk}_i, \mathsf{ske.tk}_i) \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk})$ for every $i \in [q]$ and sends $(\mathsf{ske}.dk_i, \mathsf{ske.tk}_i)_{i\in[q]}$ to $\mathcal{R}$.
4. $\mathcal{R}$ computes $\widetilde{dk}_1, \ldots, \widetilde{dk}_q$ as in Step 2. of $\mathsf{Hyb}_0^{\mathsf{coin}}$, except that the received values $(\mathsf{ske}.dk_i)_{i\in[q]}$ are used instead of the original ones.

5. $\mathcal{R}$ simulates the access to $O_{\mathcal{V}rfy}(i, \widetilde{dk})$ for $\mathcal{D}$ as follows:

   $O_{\mathcal{V}rfy}(i, \widetilde{dk})$:

   (a) Perform Step 3.(a)-3.(f) of $\mathsf{Hyb}_0^{\mathsf{coin}}$ to obtain $\mathsf{ske}.dk'$, but using the received value $\mathsf{ske.tk}_i$ instead of the original one.

   (b) Set $d := \top$ if $\top = \mathsf{SKE\text{-}CR\text{-}SKL}.O_{\mathcal{V}rfy}(i, \mathsf{ske}.dk')$ and set $d := \bot$ otherwise. It returns d to $\mathcal{D}$. Finally, if $V_i = \bot$ and $d = \top$, it updates $V_i = \top$.

6. $\mathcal{R}$ sends $(\mathsf{ske.m}_0^*, \mathsf{ske.m}_1^*) := (0^\lambda, \mathsf{lock})$ to $\mathcal{CH}$ and receives $\mathsf{ske.ct}^* \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ske.msk}, \mathsf{ske.m}_b^*)$.

7. $\mathcal{D}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{R}$. If $V_i = \bot$ for some $i \in [q]$, $\mathcal{R}$ outputs 0. Otherwise, $\mathcal{R}$ generates $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{ske.ct}^*], \mathsf{lock}, 0)$ and $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, \widetilde{C}^*, \mathsf{m}_{\mathsf{coin}}^*)$ and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{D}$.

8. $\mathcal{D}$ outputs a bit b'. $\mathcal{R}$ outputs b' and $\mathcal{CH}$ outputs b' as the final output of the experiment.

It is easy to see that the view of $\mathcal{D}$ is the same as that in $\mathsf{Hyb}_2^{\mathsf{coin}}$ when lock is encrypted in $\mathsf{ske.ct}^*$ and that of $\mathsf{Hyb}_1^{\mathsf{coin}}$ when 0^λ is encrypted. Moreover, for $\mathcal{D}$ to distinguish between the two hybrids, it must be the case that $V_i = \top$ for all $i \in [q]$, which directly implies that the q analogous values checked by $\mathsf{SKE\text{-}CR\text{-}SKL}.O_{\mathcal{V}rfy}$ must also be $\top$. Consequently, $\mathcal{R}$ breaks the OT-IND-KLA security of SKE-CR-SKL. Therefore, it must be that $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$.

$\mathsf{Hyb}_3^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_2^{\mathsf{coin}}$ except that dk_i is generated as follows for every $i \in [q]$. (The difference is colored in red.)

- Generate $(\mathsf{ske}.dk_i, \mathsf{ske.vk}_i, \mathsf{ske.tk}_i) \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{ske.msk})$. We denote the register holding $\mathsf{ske}.dk_i$ as $\mathsf{SKE.DK_i}$.
- Prepare a register $\mathsf{ABE.R_i}$ that is initialized to $|0\rangle_{\mathsf{ABE.R_i}}$.
- Apply the map $|u\rangle_{\mathsf{SKE.DK_i}} |\beta\rangle_{\mathsf{ABE.R_i}} \to |u\rangle_{\mathsf{SKE.DK_i}} \left|\beta \oplus R(\widetilde{C}^*, u)\right\rangle_{\mathsf{ABE.R_i}}$ to the registers $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.R_i}$. (Note that we can generate $\widetilde{C}^*$ at the beginning of the game.)
- Measure $\mathsf{ABE.R_i}$ in the computational basis and set $dk_i := \bot$ if the result is 0. Otherwise, go to the next step.
- Prepare a register $\mathsf{ABE.SK_i}$ that is initialized to $|0 \cdots 0\rangle_{\mathsf{ABE.SK_i}}$.
- Sample explicit randomness $\mathsf{k}_i \leftarrow \{0,1\}^\lambda$.
- To registers $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.SK_i}$, apply the map

$$|u\rangle_{\mathsf{SKE.DK_i}} |v\rangle_{\mathsf{ABE.SK_i}} \to |u\rangle_{\mathsf{SKE.DK_i}} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k}_i)\rangle_{\mathsf{ABE.SK_i}}$$

  and obtain dk_i over the registers $\mathsf{SKE.DK_i}$ and $\mathsf{ABE.SK_i}$.

From the decryption correctness of SKE-CR-SKL, the added procedure that checks $R(\widetilde{C}^*, u)$ in superposition does not affect the final state dk_i with overwhelming probability since $R(\widetilde{C}^*, u) = 1$ in this hybrid for any u that appears in $\mathsf{ske}.dk_i$ when describing it in the computational basis. Therefore, we have $\mathsf{Hyb}_2^{\mathsf{coin}} \approx \mathsf{Hyb}_3^{\mathsf{coin}}$.

$\mathsf{Hyb}_4^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_3^{\mathsf{coin}}$ except that the oracle $O_{\mathcal{V}rfy}$ behaves as follows. (The difference is colored in red.)

$O_{\mathcal{V}rfy}(i, \widetilde{dk})$**:** It computes d as follows.

(a) Let the register holding $\widetilde{dk}$ be $\mathsf{SKE.DK_i} \otimes \mathsf{ABE.SK_i}$.

(b) Prepare a register $\mathsf{SKE.KT_i}$ that is initialized to $|0\rangle_{\mathsf{SKE.KT_i}}$.

(c) To $\mathsf{SKE.DK_i}$ and $\mathsf{SKE.KT_i}$, apply the map

$$|u\rangle_{\mathsf{SKE.DK_i}} |\beta\rangle_{\mathsf{SKE.KT_i}} \to |u\rangle_{\mathsf{SKE.DK_i}} |\beta \oplus \mathsf{SKE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{ske.tk}_i, u)\rangle_{\mathsf{SKE.KT_i}}.$$

(d) Measure $\mathsf{SKE.KT}_i$ in the computational basis and set $d := \bot$ if the result is 0. Otherwise, go to the next step.

(e) Prepare a register $\mathsf{ABE.R}_i$ that is initialized to $|0\rangle_{\mathsf{ABE.R}_i}$.

(f) Apply the map $|u\rangle_{\mathsf{SKE.DK}_i} |\beta\rangle_{\mathsf{ABE.R}_i} \to |u\rangle_{\mathsf{SKE.DK}_i} \left|\beta \oplus R(\widetilde{C}^*, u)\right\rangle_{\mathsf{ABE.R}_i}$ to the registers $\mathsf{SKE.DK}_i$ and $\mathsf{ABE.R}_i$.

(g) Measure $\mathsf{ABE.R}_i$ in the computational basis and set $d := \bot$ if the result is 0. Otherwise, go to the next step.

(h) To $\mathsf{SKE.DK}_i$ and $\mathsf{ABE.SK}_i$, apply the map

$$|u\rangle_{\mathsf{SKE.DK}_i} |v\rangle_{\mathsf{ABE.SK}_i} \to |u\rangle_{\mathsf{SKE.DK}_i} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, u, \mathsf{k}_i)\rangle_{\mathsf{ABE.SK}_i}.$$

(i) Trace out the register $\mathsf{ABE.SK}_i$ and obtain $\mathsf{ske}.\widetilde{dk}'$ over $\mathsf{SKE.DK}_i$.

(j) Set $d := \top$ if $\top = \mathsf{SKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{ske.vk}_i, \mathsf{ske}.\widetilde{dk}')$ and set $d := \bot$ otherwise. Return d to $\mathcal{A}$. Finally, if $V_i = \bot$ and $d = \top$, update $V_i := \top$.

Suppose there exists a QPT distinguisher $\mathcal{D}$ that has non-negligible advantage in distinguishing $\mathsf{Hyb}_3^{\mathsf{coin}}$ and $\mathsf{Hyb}_4^{\mathsf{coin}}$. Let $\mathcal{D}$ make $q = \mathrm{poly}(\lambda)$ many queries to the oracle $O_{\mathcal{V}rfy}(\cdot,\cdot)$. We will now consider the following QPT algorithm $\mathcal{A}_{\mathsf{o2h}}$ with access to an oracle $O_{\mathcal{K}\mathcal{G}}$ and an oracle $\mathcal{O}$ that runs $\mathcal{D}$ as follows:

$\underline{\mathcal{A}_{\mathsf{o2h}}^{O_{\mathcal{K}\mathcal{G}},\mathcal{O}}(\mathsf{abe.pk}, \mathsf{ske.msk})}$:

1. $\mathcal{A}_{\mathsf{o2h}}$ initializes $\mathcal{D}$ with input $\mathsf{ek} = \mathsf{abe.pk}$.
2. When $\mathcal{D}$ requests q decryption keys, $\mathcal{A}_{\mathsf{o2h}}$ queries $O_{\mathcal{K}\mathcal{G}}$ on input q and receives the response $(\{\widetilde{dk}_i\}_{i\in[q]}, \{\mathsf{vk}_i\}_{i\in[q]})$. It forwards $\{\widetilde{dk}_i\}_{i\in[q]}$ to $\mathcal{D}$.
3. $\mathcal{A}_{\mathsf{o2h}}$ simulates the access of $O_{\mathcal{V}rfy}$ for $\mathcal{D}$ as follows:

   $\underline{O_{\mathcal{V}rfy}(y, \widetilde{dk})}$:

   (a) Execute Steps (a)-(e) of $O_{\mathcal{V}rfy}$ as in $\mathsf{Hyb}_4^{\mathsf{coin}}$.

   (b) To $\mathsf{SKE.DK}_y$ and $\mathsf{ABE.R}_y$, apply the map $|u\rangle_{\mathsf{SKE.DK}_y} |\beta\rangle_{\mathsf{ABE.R}_y} \to |u\rangle_{\mathsf{SKE.DK}_y} |\beta \oplus \mathcal{O}(u)\rangle_{\mathsf{ABE.R}_y}$.

   (c) Execute Steps (g)-(j) of $O_{\mathcal{V}rfy}$ as in $\mathsf{Hyb}_4^{\mathsf{coin}}$.
4. $\mathcal{D}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{A}_{\mathsf{o2h}}$. If $V_i = \bot$ for some $i \in [q]$, $\mathcal{A}_{\mathsf{o2h}}$ outputs 0. Otherwise, $\mathcal{A}_{\mathsf{o2h}}$ generates $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{ske.ct}^*], \mathsf{lock}, 0)$, where $\mathsf{ske.ct}^* \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Enc}(\mathsf{ske.msk}, \mathsf{lock})$. It then generates $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, \widetilde{C}^*, \mathsf{m}^*_{\mathsf{coin}})$ and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{D}$.
5. $\mathcal{D}$ outputs a guess b'. $\mathcal{A}_{\mathsf{o2h}}$ outputs b'.

Let H be an oracle that for every input u, outputs 1. Consider now the extractor $\mathcal{B}_{\mathsf{o2h}}^{O_{\mathcal{K}\mathcal{G}},H}$ as specified by the O2H Lemma (Lemma 3.2). We will now construct a reduction $\mathcal{R}$ that runs $\mathcal{B}_{\mathsf{o2h}}$ by simulating the oracles $O_{\mathcal{K}\mathcal{G}}$ and H for $\mathcal{B}_{\mathsf{o2h}}$, and breaks the key-testability of the $\mathsf{SKE\text{-}CR\text{-}SKL}$ scheme.

Execution of $\mathcal{R}$ in $\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{R}}(1^\lambda)$:

1. The challenger $\mathcal{C}h$ runs $\mathsf{ske.msk} \leftarrow \mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{R}$ with input $\mathsf{ske.msk}$.
2. $\mathcal{R}$ samples $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and initializes $\mathcal{B}_{\mathsf{o2h}}$ with the input $(\mathsf{abe.pk}, \mathsf{ske.msk})$.
3. When $\mathcal{B}_{\mathsf{o2h}}$ queries input q to $O_{\mathcal{K}\mathcal{G}}$, $\mathcal{R}$ generates the decryption-keys $\widetilde{dk}_1, \ldots, \widetilde{dk}_q$ in the same way as in $\mathsf{Hyb}_3^{\mathsf{coin}}$ and sends them to $\mathcal{B}_{\mathsf{o2h}}$.
4. When $\mathcal{B}_{\mathsf{o2h}}$ queries an input u to H, $\mathcal{R}$ responds with 1.
5. $\mathcal{B}_{\mathsf{o2h}}$ outputs measured index y and measurement outcome dk. $\mathcal{R}$ sends $(y, \mathsf{dk}, \mathsf{lock})$ to $\mathcal{C}h$.

We will now claim that with non-negligible probability, $\mathcal{R}$ obtains values dk and y such that $R(\widetilde{C}^*, \mathsf{dk}) = 0$. By the definition of R and $\widetilde{C}^*$ and the decryption correctness of $\mathsf{SKE\text{-}CR\text{-}SKL}$, this will imply that $\mathsf{SKE\text{-}CR\text{-}SKL.CDec}(\mathsf{dk}, \mathsf{ske.ct}^\star) \neq \mathsf{lock}$. Moreover, $\mathsf{KeyTest}(\mathsf{ske.tk}_y, \mathsf{dk})$ also holds. Consequently, this will imply $\mathcal{R}$ breaks the key-testability of $\mathsf{SKE\text{-}CR\text{-}SKL}$. To prove this, we will rely on the One-Way to Hiding (O2H) Lemma (Lemma 3.2). Consider an oracle G which takes as input u and outputs $R(\widetilde{C}^*, u)$ and an oracle H which takes as input u and outputs 1. Notice that if the oracle $\mathcal{O} = G$, then the view of $\mathcal{D}$ as run by $\mathcal{A}_{\mathsf{o2h}}$ is the same as in $\mathsf{Hyb}_4^{\mathsf{coin}}$, while if $\mathcal{O} = H$, the view of $\mathcal{D}$ is the same as in $\mathsf{Hyb}_3^{\mathsf{coin}}$. By the O2H Lemma, we have the following, where $z = (\mathsf{abe.pk}, \mathsf{ske.msk})$.

$$\left|\Pr\left[\mathcal{A}_{\mathsf{o2h}}^{O_{\mathcal{KG}}, H}(z) = 1\right] - \Pr\left[\mathcal{A}_{\mathsf{o2h}}^{O_{\mathcal{KG}}, G}(z) = 1\right]\right| \leq 2q \cdot \sqrt{\Pr\left[\mathcal{B}_{\mathsf{o2h}}^{O_{\mathcal{KG}}, H}(z) \in S\right]} \ .$$

where S is a set where the oracles H and G differ, which happens only for inputs u s.t. $R(\widetilde{C}^*, u) = 0$. Since $\mathcal{R}$ obtains dk and y as the output of $\mathcal{B}_{\mathsf{o2h}}^{O_{\mathcal{KG}}, H}(z)$, the argument follows that $\mathsf{Hyb}_3^{\mathsf{coin}} \approx \mathsf{Hyb}_4^{\mathsf{coin}}$.

$\mathsf{Hyb}_5^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_4^{\mathsf{coin}}$ except that $\mathsf{ct}^* := \mathsf{abe.ct}^*$ is generated as $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, \widetilde{C}^*, 0^{\ell_{\mathsf{m}}})$.

The view of $\mathcal{A}$ in $\mathsf{Hyb}_4^{\mathsf{coin}}$ and $\mathsf{Hyb}_5^{\mathsf{coin}}$ can be simulated with $\mathsf{abe.pk}$ and the access to the quantum key generation oracle O_{qkg}. This is because before $\mathsf{ABE.KG}$ is required to be applied in both the generation of $\{\mathsf{dk}_i\}_{i\in[q]}$ and to compute the responses of $O_{\mathcal{V}\mathit{rfy}}$, the relation check $R(\widetilde{C}^*, u)$ is already applied in superposition. Thus, we have $\mathsf{Hyb}_4^{\mathsf{coin}} \approx \mathsf{Hyb}_5^{\mathsf{coin}}$.

Lastly, Hyb_5^0 and Hyb_5^1 are exactly the same experiment and thus we have $\left|\Pr\left[\mathsf{Hyb}_5^0 = 1\right] - \Pr\left[\mathsf{Hyb}_5^1 = 1\right]\right| = \mathsf{negl}(\lambda)$. Then, from the above arguments, we obtain

$$\begin{aligned}&\left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 0) = 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 1) = 1\right]\right| \\ =&\left|\Pr\left[\mathsf{Hyb}_0^0 = 1\right] - \Pr\left[\mathsf{Hyb}_0^1 = 1\right]\right| \leq \mathsf{negl}(\lambda).\end{aligned}$$

This completes the proof. □

Given the fact that SKE-CR-SKL with Key-Testability (implied by BB84-based SKE-CD and OWFs), Compute-and-Compare Obfuscation, and Ciphertext-Policy ABE for General Circuits are all implied by the LWE assumption, we state the following theorem:

Theorem 6.1. *There exists a PKE-CR-SKL scheme satisfying IND-KLA security, assuming the polynomial hardness of the LWE assumption.*

7 ABE-CR-SKL from LWE

In this section, we show how to achieve ABE-CR-SKL from the LWE assumption. To this end, we also introduce SKFE-CR-SKL with classical decryption and key-testability. First, we recall the standard SKFE.

Definition 7.1 (Secret-Key Functional Encryption). *An SKFE scheme* SKFE *for the functionality* $F : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ *is a tuple of four PPT algorithms* $(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$.

$\mathsf{Setup}(1^\lambda) \to \mathsf{msk}$**:** *The setup algorithm takes a security parameter* 1^λ*, and outputs a master secret key* msk.

$\mathsf{KG}(\mathsf{msk}, y) \to \mathsf{sk}_y$**:** *The key generation algorithm takes a master secret key* msk *and a function* $y \in \mathcal{Y}$*, and outputs a functional decryption key* sk_y.

$\mathsf{Enc}(\mathsf{msk}, x) \to \mathsf{ct}$**:** *The encryption algorithm takes a master secret key* msk *and a plaintext* $x \in \mathcal{X}$*, and outputs a ciphertext* ct.

$\mathsf{Dec}(\mathsf{sk}_y, \mathsf{ct}) \to z$**:** *The decryption algorithm takes a functional decryption key* sk_y *and a ciphertext* ct*, and outputs* $z \in \{\bot\} \cup \mathcal{Z}$*.*

Correctness: *We require that for every* $x \in \mathcal{X}$ *and* $y \in \mathcal{Y}$*, we have that*

$$\Pr\left[\mathsf{Dec}(\mathsf{sk}_y, \mathsf{ct}) = F(x,y) \ : \ \begin{array}{l} \mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda) \\ \mathsf{sk}_y \leftarrow \mathsf{KG}(\mathsf{msk}, y) \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, x) \end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Definition 7.2 (Selective Single-Ciphertext Security). *We formalize the experiment* $\mathsf{Exp}^{\mathsf{adp\text{-}ind}}_{\mathsf{SKFE},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger for an SKFE scheme for the functionality* $F : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ *as follows:*

1. *Initialized with* 1^λ*,* $\mathcal{A}$ *outputs* (x_0^*, x_1^*)*. The challenger* Ch *runs* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$ *and sends* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{msk}, x^*_{\mathsf{coin}})$ *to* $\mathcal{A}$*.*
2. $\mathcal{A}$ *can get access to the following oracle.*

   $O_{\mathcal{KG}}(y)$**:** *Given* y*, if* $F(x_0^*, y) \neq F(x_1^*, y)$*, returns* $\bot$*. Otherwise, it returns* $\mathsf{sk}_y \leftarrow \mathsf{KG}(\mathsf{msk}, y)$*.*
3. $\mathcal{A}$ *outputs a guess* coin' *for* coin*.* $\mathcal{C}h$ *outputs* coin' *as the final output of the experiment.*

We say that SKFE *satisfies selective single-ciphertext security if, for any QPT* $\mathcal{A}$*, it holds that*

$$\begin{aligned} &\mathsf{Adv}^{\mathsf{sel\text{-}1ct}}_{\mathsf{SKFE},\mathcal{A}}(1^\lambda) := \\ &\left|\Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}1ct}}_{\mathsf{SKFE},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}1ct}}_{\mathsf{SKFE},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda). \end{aligned}$$

Theorem 7.3 ([GVW12]). *Assuming the existence of OWFs, there exists selective single-ciphertext secure SKFE.*

7.1 Definitions of SKFE-CR-SKL

We consider the classical decryption property and key-testability for SKFE-CR-SKL as in the case of SKE-CR-SKL.

Definition 7.4 (SKFE-CR-SKL). *An SKFE-CR-SKL scheme* SKFE-CR-SKL *for the functionality* $F : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ *is a tuple of five algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{D}ec, \mathcal{V}rfy)$*.*

$\mathsf{Setup}(1^\lambda) \to \mathsf{msk}$**:** *The setup algorithm takes a security parameter* 1^λ *and a master secret key* msk*.*

$\mathcal{KG}(\mathsf{msk}, y) \to (\mathit{sk}_y, \mathsf{vk}, \mathsf{tk})$**:** *The key generation algorithm takes a master secret key* msk *and a string* $y \in \mathcal{Y}$*, and outputs a functional secret key* sk_y*, a certificate verification key* vk*, and a testing key* tk*.*

$\mathsf{Enc}(\mathsf{msk}, x) \to \mathsf{ct}$**:** *The encryption algorithm takes a master secret key* msk *and a string* $x \in \mathcal{X}$*, and outputs a ciphertext* ct*.*

$\mathcal{D}ec(\mathit{sk}_y, \mathsf{ct}) \to z$**:** *The decryption algorithm takes a functional secret key* sk_y *and a ciphertext* ct*, and outputs a value* $z \in \mathcal{Z} \cup \{\bot\}$*.*

$\mathcal{V}rfy(\mathsf{vk}, \widetilde{\mathit{sk}}_y) \to \top / \bot$**:** *The verification algorithm takes a verification key* vk *and a (possibly malformed) functional secret key* $\widetilde{\mathit{sk}}_y$*, and outputs* $\top$ *or* $\bot$*.*

Decryption correctness: *For all* $x \in \mathcal{X}$ *and* $y \in \mathcal{Y}$*, we have*

$$\Pr\left[\mathcal{D}ec(\mathit{sk}_y, \mathsf{ct}) = F(x,y) \ : \ \begin{array}{l} \mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{sk}_y, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y) \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, x) \end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *We have*

$$\Pr\left[\mathcal{Vrfy}(\mathsf{vk}, sk_y) = \top \ : \ \begin{array}{l}\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)\\ (sk_y, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Definition 7.5 (Classical Decryption Property). *We say that an SKFE-CR-SKL scheme* $\mathsf{SKFE\text{-}CR\text{-}SKL} = (\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$ *has the classical decryption property if there exists a deterministic polynomial time algorithm* CDec *such that given* sk_y *in the register* SK *and ciphertext* ct*,* $\mathcal{Dec}$ *applies the map* $|u\rangle_{\mathsf{SK}} |v\rangle_{\mathsf{OUT}} \to |u\rangle_{\mathsf{SK}} |v \oplus \mathsf{CDec}(u, \mathsf{ct})\rangle_{\mathsf{OUT}}$ *and outputs the measurement result of the register* OUT *in the computational basis, where* OUT *is initialized to* $|0 \cdots 0\rangle_{\mathsf{OUT}}$.

Definition 7.6 (Key-Testability). *We say that an SKFE-CR-SKL scheme* SKFE-CR-SKL *with the classical decryption property satisfies key testability, if there exists a classical deterministic algorithm* KeyTest *that satisfies the following conditions:*

- ***Syntax:*** KeyTest *takes as input a testing key* tk *and a classical string* sk *as input. It outputs* 0 *or* 1.
- ***Correctness:*** *Let* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$ *and* $(sk_y, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$ *where* y *is a string. We denote the register holding* sk_y *as* SK*. Let* KT *be a register that is initialized to* $|0\rangle_{\mathsf{KT}}$*. If we apply the map* $|u\rangle_{\mathsf{SK}} |\beta\rangle_{\mathsf{KT}} \to |u\rangle_{\mathsf{SK}} |\beta \oplus \mathsf{KeyTest}(\mathsf{tk}, u)\rangle_{\mathsf{KT}}$ *to the registers* SK *and* KT *and then measure* KT *in the computational basis, we obtain* 1 *with overwhelming probability.*
- ***Security:*** *Consider the following experiment* $\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda)$.

  1. *The challenger* $\mathcal{Ch}$ *runs* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$ *and initializes* $\mathcal{A}$ *with input* msk.
  2. $\mathcal{A}$ *can get access to the following oracle, where the list* $L_{\mathcal{KG}}$ *used by the oracles is initialized to an empty list.*

     $O_{\mathcal{KG}}(y)$**:** *Given* y*, it finds an entry of the form* (y, tk) *from* $L_{\mathcal{KG}}$*. If there is such an entry, it returns* $\perp$*. Otherwise, it generates* $(sk_y, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$*, sends* $(sk_y, \mathsf{vk}, \mathsf{tk})$ *to* $\mathcal{A}$*, and adds* (y, tk) *to* $L_{\mathcal{KG}}$.
  3. $\mathcal{A}$ *sends a tuple of classical strings* (y, sk, x) *to* $\mathcal{Ch}$*.* $\mathcal{Ch}$ *outputs* $\perp$ *if there is no entry of the form* (y, tk) *in* $L_{\mathcal{KG}}$ *for some* tk*. Otherwise,* $\mathcal{Ch}$ *generates* $\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, x)$*, and outputs* $\top$ *if* $\mathsf{KeyTest}(\mathsf{tk}, \mathsf{sk}) = 1$ *and* $\mathsf{CDec}(\mathsf{sk}, \mathsf{ct}) \neq F(x, y)$*, and outputs* $\perp$ *otherwise.*

  For all QPT $\mathcal{A}$*, the following must hold:*

$$\mathsf{Adv}^{\mathsf{key\text{-}test}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \Pr\left[\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) \to \top\right] \leq \mathsf{negl}(\lambda).$$

Definition 7.7 (Selective Single-Ciphertext KLA Security). *We say that an SKFE-CR-SKL scheme* SKFE-CR-SKL *is selective single-ciphertext secure, if it satisfies the following requirement, formalized from the experiment* $\mathsf{Exp}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger:*

1. *Initialized with* 1^λ*,* $\mathcal{A}$ *outputs* (x_0^*, x_1^*)*. The challenger* $\mathcal{Ch}$ *runs* $\mathsf{msk} \leftarrow \mathsf{Setup}(1^\lambda)$.
2. $\mathcal{A}$ *can get access to the following (stateful) oracles, where the list* $L_{\mathcal{KG}}$ *used by the oracles is initialized to an empty list:*

   $O_{\mathcal{KG}}(y)$**:** *Given* y*, it finds an entry of the form* (y, vk, V) *from* $L_{\mathcal{KG}}$*. If there is such an entry, it returns* $\perp$*. Otherwise, it generates* $(sk, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$*, sends* sk *and* tk *to* $\mathcal{A}$*, and adds* $(y, \mathsf{vk}, \perp)$ *to* $L_{\mathcal{KG}}$.

   $O_{\mathcal{Vrfy}}(y, \widetilde{sk})$**:** *Given* $(y, \widetilde{sk})$*, it finds an entry* (y, vk, V) *from* $L_{\mathcal{KG}}$*. (If there is no such entry, it returns* $\perp$*.) It then runs* $d \leftarrow \mathcal{Vrfy}(\mathsf{vk}, \widetilde{sk})$ *and returns* d *to* $\mathcal{A}$*. If* $V = \perp$*, it updates the entry into* (y, vk, d).

3. *$\mathcal{A}$ requests the challenge ciphertext. If there exists an entry (y, vk, V) in $L_{\mathcal{KG}}$ such that $F(x_0^*, y) \neq F(x_1^*, y)$ and $V = \bot$, $\mathcal{Ch}$ outputs 0 as the final output of this experiment. Otherwise, $\mathcal{Ch}$ generates $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{msk}, x^*_{\mathsf{coin}})$ and sends ct^* to $\mathcal{A}$.*

4. *$\mathcal{A}$ continues to make queries to $O_{\mathcal{KG}}$. However, $\mathcal{A}$ is not allowed to send y such that $F(x_0^*, y) \neq F(x_1^*, y)$ to $O_{\mathcal{KG}}$.*

5. *$\mathcal{A}$ outputs a guess coin' for coin. $\mathcal{Ch}$ outputs coin' as the final output of the experiment.*

For any QPT $\mathcal{A}$, it holds that

$$\mathsf{Adv}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

In Appendix C, we prove the following theorem:

Theorem 7.8. *Assuming the existence of a BB84-based SKE-CD scheme and the existence of OWFs, there exists a selective single-ciphertext KLA secure SKFE-CR-SKL scheme satisfying the key-testability property.*

7.2 Definitions of ABE-CR-SKL

In this section, we recall definitions of ABE-CR-SKL by Agrawal et al. [AKN+23].

Definition 7.9 (ABE-CR-SKL). *An ABE-CR-SKL scheme* ABE-CR-SKL *is a tuple of five algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$*. Below, let $\mathcal{X} = \{\mathcal{X}_\lambda\}_\lambda$, $\mathcal{Y} = \{\mathcal{Y}_\lambda\}_\lambda$, and $R = \{R_\lambda : \mathcal{X}_\lambda \times \mathcal{Y}_\lambda \to \{0,1\}\}_\lambda$ be the ciphertext attribute space, the key attribute space, and the associated relation of* ABE-CR-SKL*, respectively. Let $\mathcal{M}$ denote the message space.*

$\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter 1^λ, and outputs a public key* pk *and master secret key* msk*.*

$\mathcal{KG}(\mathsf{msk}, y) \to (\mathit{sk}_y, \mathsf{vk})$**:** *The key generation algorithm takes a master secret key* msk *and a key attribute $y \in \mathcal{Y}$, and outputs a user secret key sk_y and a verification key* vk*.*

$\mathsf{Enc}(\mathsf{pk}, x, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes a public key* pk*, a ciphertext attribute $x \in \mathcal{X}$, and a plaintext $\mathsf{m} \in \mathcal{M}$, and outputs a ciphertext* ct*.*

$\mathcal{Dec}(\mathit{sk}_y, \mathsf{ct}) \to \mathsf{m}'$**:** *The decryption algorithm takes a user secret key sk_y and a ciphertext* ct*. It outputs a value $\mathsf{m}' \in \{\bot\} \cup \mathcal{M}$.*

$\mathcal{Vrfy}(\mathsf{vk}, \mathit{sk}') \to \top / \bot$**:** *The verification algorithm takes a verification key* vk *and a quantum state sk', and outputs $\top$ or $\bot$.*

Decryption correctness: *For every $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ satisfying $R(x, y) = 0$ and $\mathsf{m} \in \mathcal{M}$, we have*

$$\Pr\left[\mathcal{Dec}(\mathit{sk}_y, \mathsf{ct}) = \mathsf{m} \;\middle|\; \begin{array}{l}(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{sk}_y, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y) \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{pk}, x, \mathsf{m})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *For every $y \in \mathcal{Y}$, we have*

$$\Pr\left[\mathcal{Vrfy}(\mathsf{vk}, \mathit{sk}_y) = \top \;\middle|\; \begin{array}{l}(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{sk}_y, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Definition 7.10 (Adaptive IND-KLA Security). *We say that an ABE-CR-SKL scheme* ABE-CR-SKL *for relation $R : \mathcal{X} \times \mathcal{Y} \to \{0,1\}$ is adaptively IND-KLA secure, if it satisfies the following requirement, formalized from the experiment $\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$ between an adversary $\mathcal{A}$ and a challenger $\mathcal{Ch}$:*

1. *At the beginning, $\mathcal{Ch}$ runs $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ and initializes the list $L_{\mathcal{KG}}$ to be an empty set. Throughout the experiment, $\mathcal{A}$ can access the following oracles.*

   $O_{\mathcal{KG}}(y)$**:** *Given y, it finds an entry of the form (y, vk, V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it generates $(sk_y, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$, sends sk_y to $\mathcal{A}$, and adds $(y, \mathsf{vk}, \bot)$ to $L_{\mathcal{KG}}$.*

   $O_{\mathcal{Vrfy}}(y, \widetilde{sk})$**:** *Given $(y, \widetilde{sk})$, it finds an entry (y, vk, V) from $L_{\mathcal{KG}}$. (If there is no such entry, it returns $\bot$.) It then runs $d := \mathcal{Vrfy}(\mathsf{vk}, \widetilde{sk})$ and returns d to $\mathcal{A}$. If $V = \bot$, it updates the entry into (y, vk, d).*

2. *When $\mathcal{A}$ sends $(x^*, \mathsf{m}_0, \mathsf{m}_1)$ to $\mathcal{Ch}$, it checks that for every entry (y, vk, V) in $L_{\mathcal{KG}}$ such that $R(x^*, y) = 0$, it holds that $V = \top$. If so, $\mathcal{Ch}$ generates $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{pk}, x^*, \mathsf{m}_{\mathsf{coin}})$ and sends ct^* to $\mathcal{A}$. Otherwise, it outputs 0.*

3. *$\mathcal{A}$ continues to make queries to $O_{\mathcal{KG}}(\cdot)$ and $O_{\mathcal{Vrfy}}(\cdot, \cdot)$. However, $\mathcal{A}$ is not allowed to send a key attribute y such that $R(x^*, y) = 0$ to $O_{\mathcal{KG}}$.*

4. *$\mathcal{A}$ outputs a guess coin' for coin. $\mathcal{Ch}$ outputs coin' as the final output of the experiment.*

For any QPT $\mathcal{A}$, it holds that

$$\mathsf{Adv}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

Remark 7.11. Although we can handle the situation where multiple keys for the same attribute y are generated using an index management such as $(y, 1, vk_1, V_1)$, $(y, 2, vk_2, V_2)$, we use the simplified definition as Agrawal et al. [AKN+23] did.

We also consider relaxed versions of the above security notion.

Definition 7.12 (Selective IND-KLA Security). *We consider the same security game as that for adaptive IND-KLA security except that the adversary $\mathcal{A}$ should declare its target x^* at the beginning of the game (even before it is given pk).*

We then define the advantage $\mathsf{Adv}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda)$ for the selective case similarly. We say $\mathsf{ABE\text{-}CR\text{-}SKL}$ is selectively IND-KLA secure if for any QPT adversary $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda)$ is negligible.

7.3 Construction

We construct an ABE-CR-SKL scheme $\mathsf{ABE\text{-}CR\text{-}SKL} = \mathsf{ABE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$ for the relation R with the message space $\mathcal{M} := \{0,1\}^{\ell_\mathsf{m}}$ using the following building blocks.

- ABE scheme $\mathsf{ABE} = \mathsf{ABE}.(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$ for the following relation R'.

  $R'(x', y')$**:** Interpret $x' := x \| C$ and $y' := y \| z$, where C is a circuit. Then, output 0 if $R(x, y) = 0$ and $C(z) = \bot$, and otherwise output 1.

- Compute-and-Compare Obfuscation $\mathsf{CC.Obf}$ with the simulator $\mathsf{CC.Sim}$.

- SKFE-CR-SKL scheme with key-testability $\mathsf{SKFE\text{-}CR\text{-}SKL} = \mathsf{SKFE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy}, \mathsf{KeyTest})$ for the following functionality F. It also has the classical decryption algorithm $\mathsf{SKFE\text{-}CR\text{-}SKL.CDec}$.

  $F(\widetilde{x}, \widetilde{y})$**:** Interpret $\widetilde{x} := x \| z$ and $\widetilde{y} := y$. Then, output z if $R(x, y) = 0$, and otherwise output $\bot$.

The construction is as follows.

$\mathsf{ABE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$**:**

- Generate $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$.
- Generate $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$.

- Output $\mathsf{pk} := \mathsf{abe.pk}$ and $\mathsf{msk} := (\mathsf{abe.msk}, \mathsf{skfe.msk})$.

$\mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk}, y)$:

- Parse $\mathsf{msk} = (\mathsf{abe.msk}, \mathsf{skfe.msk})$.
- Generate $(\mathsf{skfe}.s\mathcal{k}, \mathsf{skfe.vk}, \mathsf{skfe.tk}) \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{skfe.msk}, y)$. We denote the register holding $\mathsf{skfe}.s\mathcal{k}$ as $\mathsf{SKFE.SK}$.
- Sample explicit randomness $\mathsf{k} \leftarrow \{0,1\}^\lambda$.
- Prepare a register $\mathsf{ABE.SK}$ that is initialized to $|0\cdots 0\rangle_{\mathsf{ABE.SK}}$.
- To registers $\mathsf{SKFE.SK}$ and $\mathsf{ABE.SK}$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}} |v\rangle_{\mathsf{ABE.SK}} \to |u\rangle_{\mathsf{SKFE.SK}} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y\|u, \mathsf{k})\rangle_{\mathsf{ABE.SK}}$$

  and obtain $s\mathcal{k}$ over the registers $\mathsf{SKFE.SK}$ and $\mathsf{ABE.SK}$.
- Output $s\mathcal{k}$ and $\mathsf{vk} := (y, \mathsf{abe.msk}, \mathsf{skfe.vk}, \mathsf{skfe.tk}, \mathsf{k})$.

$\mathsf{ABE\text{-}CR\text{-}SKL.Enc}(\mathsf{pk}, x, \mathsf{m})$:

- Parse $\mathsf{pk} = \mathsf{abe.pk}$.
- Generate $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$, where pp_D consists of circuit parameters of D defined in the security proof.
- Generate $\mathsf{abe.ct} \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x\|\widetilde{C}, \mathsf{m})$.
- Output $\mathsf{ct} := \mathsf{abe.ct}$.

$\mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{D}ec(s\mathcal{k}, \mathsf{ct})$:

- Parse $\mathsf{ct} = \mathsf{abe.ct}$. We denote the register holding $s\mathcal{k}$ as $\mathsf{SKFE.SK} \otimes \mathsf{ABE.SK}$.
- Prepare a register MSG that is initialized to $|0\cdots 0\rangle_{\mathsf{MSG}}$
- Apply the map $|v\rangle_{\mathsf{ABE.SK}} |w\rangle_{\mathsf{MSG}} \to |v\rangle_{\mathsf{ABE.SK}} |w \oplus \mathsf{ABE.Dec}(v, \mathsf{abe.ct})\rangle_{\mathsf{MSG}}$ to the registers $\mathsf{ABE.SK}$ and MSG.
- Measure the register MSG in the computational basis and output the result m'.

$\mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, s\mathcal{k}')$:

- Parse $\mathsf{vk} = (y, \mathsf{abe.msk}, \mathsf{skfe.vk}, \mathsf{skfe.tk}, \mathsf{k})$. We denote the register holding $s\mathcal{k}'$ as $\mathsf{SKFE.SK} \otimes \mathsf{ABE.SK}$.
- Prepare a register $\mathsf{SKFE.KT}$ that is initialized to $|0\rangle_{\mathsf{SKFE.KT}}$.
- To $\mathsf{SKFE.SK}$ and $\mathsf{SKFE.KT}$, apply the map:

$$|u\rangle_{\mathsf{SKFE.SK}} |\beta\rangle_{\mathsf{SKFE.KT}} \to |u\rangle_{\mathsf{SKFE.SK}} |\beta \oplus \mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{skfe.tk}, u)\rangle_{\mathsf{SKFE.KT}}.$$

- Measure $\mathsf{SKFE.KT}$ in the computational basis and output $\bot$ if the result is 0. Otherwise, go to the next step.
- To registers $\mathsf{SKFE.SK}$ and $\mathsf{ABE.SK}$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}} |v\rangle_{\mathsf{ABE.SK}} \to |u\rangle_{\mathsf{SKFE.SK}} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y\|u, \mathsf{k})\rangle_{\mathsf{ABE.SK}}.$$

- Trace out the register $\mathsf{ABE.SK}$ and obtain $\mathsf{skfe}.s\mathcal{k}'$ over register $\mathsf{SKFE.SK}$.
- Output $\top$ if $\top = \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{skfe.vk}, \mathsf{skfe}.s\mathcal{k}')$ and $\bot$ otherwise.

Decryption correctness: Let x and y be a ciphertext-attribute and a key-attribute, respectively, such that $R(x,y)=0$. The secret-key sk output by $\mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{KG}$ is of the form $\sum_u \alpha_u |u\rangle_{\mathsf{SKFE.SK}} \left|\mathsf{abe.sk}_{y\|u}\right\rangle_{\mathsf{ABE.SK}}$, where $\mathsf{abe.sk}_{y\|u} \leftarrow \mathsf{ABE.KG}(\mathsf{abe.msk}, y\|u, \mathsf{k})$. Let $\mathsf{ct} \leftarrow \mathsf{ABE\text{-}CR\text{-}SKL.Enc}(\mathsf{pk}, x, \mathsf{m})$, where $\mathsf{ct} = \mathsf{abe.ct} \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x\|\widetilde{C}, \mathsf{m})$ and $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$. If we apply the map

$$|v\rangle_{\mathsf{ABE.SK}} |w\rangle_{\mathsf{MSG}} \to |v\rangle_{\mathsf{ABE.SK}} |w \oplus \mathsf{ABE.Dec}(v, \mathsf{abe.ct})\rangle_{\mathsf{MSG}}$$

to $\sum_u \alpha_u |u\rangle_{\mathsf{SKFE.SK}} \left|\mathsf{abe.sk}_{y\|u}\right\rangle_{\mathsf{ABE.SK}} \otimes |0\cdots 0\rangle_{\mathsf{MSG}}$, the result is

$$\sum_u \alpha_u |u\rangle_{\mathsf{SKFE.SK}} \left|\mathsf{abe.sk}_{y\|u}\right\rangle_{\mathsf{ABE.SK}} \otimes |\mathsf{m}\rangle_{\mathsf{MSG}}$$

since $\widetilde{C}(u) = \bot$ and thus $R'(x\|\widetilde{C}, y\|u) = 0$ for every string u. Therefore, ABE-CR-SKL satisfies decryption correctness.

Verification correctness. Let y be a key attribute and $(sk, \mathsf{vk}) \leftarrow \mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk}, y)$. It is clear that the state $\mathsf{skfe}.sk'$ obtained during the computation of $\mathsf{ABE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, sk)$ is the same as $\mathsf{skfe}.sk$ generated when generating sk. Therefore, the verification correctness of ABE-CR-SKL follows from that of SKFE-CR-SKL.

7.4 Proof of Selective IND-KLA Security

Let $\mathcal{A}$ be an adversary for the selective IND-KLA security of ABE-CR-SKL. We consider the following sequence of experiments.

$\mathsf{Hyb}_0^{\mathsf{coin}}$**:** This is $\mathsf{Exp}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$.

1. $\mathcal{A}$ declares the challenge ciphertext attribute x^*. The challenger $\mathcal{C}h$ generates $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$, and sends $\mathsf{pk} := \mathsf{abe.pk}$ to $\mathcal{A}$.
2. $\mathcal{A}$ can get access to the following (stateful) oracles, where the list $L_{\mathcal{KG}}$ used by the oracles is initialized to an empty list:

   $O_{\mathcal{KG}}(y)$**:** Given y, it finds an entry of the form (y, vk_y, V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it generates sk_y, vk_y as follows.
   - Generate the keys $(\mathsf{skfe}.sk_y, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y) \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{skfe.msk}, y)$. We denote the register of $\mathsf{skfe}.sk_y$ as $\mathsf{SKFE.SK_y}$.
   - Prepare a register $\mathsf{ABE.SK_y}$ that is initialized to $|0\cdots 0\rangle_{\mathsf{ABE.SK_y}}$.
   - Sample explicit randomness $\mathsf{k}_y \leftarrow \{0,1\}^\lambda$.
   - To $\mathsf{SKFE.SK_y}$ and $\mathsf{ABE.SK_y}$, apply the map

     $$|u\rangle_{\mathsf{SKFE.SK_y}} |v\rangle_{\mathsf{ABE.SK_y}} \to |u\rangle_{\mathsf{SKFE.SK_y}} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y\|u, \mathsf{k}_y)\rangle_{\mathsf{ABE.SK_y}}$$

     and obtain sk_y over the registers $\mathsf{SKFE.SK_y}$ and $\mathsf{ABE.SK_y}$.
   - Set $\mathsf{vk}_y := (y, \mathsf{abe.msk}, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y, \mathsf{k}_y)$.

   It returns sk_y to $\mathcal{A}$ and adds the entry $(y, \mathsf{vk}_y, \bot)$ to $L_{\mathcal{KG}}$.

   $O_{\mathcal{V}rfy}(y, \widetilde{sk})$**:** It finds an entry (y, vk_y, V) from $L_{\mathcal{KG}}$. (If there is no such entry, it returns $\bot$.) It parses $\mathsf{vk}_y = (y, \mathsf{abe.msk}, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y, \mathsf{k}_y)$ and computes d as follows.
   - Let the register holding $\widetilde{sk}$ be $\mathsf{SKFE.SK_y} \otimes \mathsf{ABE.SK_y}$.
   - Prepare a register $\mathsf{SKE.KT_y}$ that is initialized to $|0\rangle_{\mathsf{SKE.KT_y}}$.

- To $\mathsf{SKFE.SK_y}$ and $\mathsf{SKE.KT_y}$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK_y}} |\beta\rangle_{\mathsf{SKE.KT_y}} \to |u\rangle_{\mathsf{SKFE.SK_y}} \left|\beta \oplus \mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{skfe.tk}_y, u)\right\rangle_{\mathsf{SKE.KT_y}}.$$

- Measure $\mathsf{SKE.KT_y}$ in the computational basis and set $d := \bot$ if the result is 0. Otherwise, go to the next step.
- To $\mathsf{SKFE.SK_y}$ and $\mathsf{ABE.SK_y}$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK_y}} |v\rangle_{\mathsf{ABE.SK_y}} \to |u\rangle_{\mathsf{SKFE.SK_y}} \left|v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y\|u, \mathsf{k}_y)\right\rangle_{\mathsf{ABE.SK_y}}.$$

- Trace out the register $\mathsf{ABE.SK_y}$ and obtain $\mathsf{skfe}.sk'$ over $\mathsf{SKFE.SK_y}$.
- Set $d := \top$ if $\top = \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{skfe.vk}_y, \mathsf{skfe}.sk')$ and set $d := \bot$ otherwise. It returns d to $\mathcal{A}$. Finally, if $V = \bot$, it updates the entry (y, vk_y, V) to (y, vk_y, d).

3. $\mathcal{A}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{CH}$. $\mathcal{CH}$ checks if for every entry (y, vk_y, V) in $L_{\mathcal{KG}}$ such that $R(x^*, y) = 0$, it holds that $V = \top$. If so, it generates $\widetilde{C}^* \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$ and $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x^*\|\widetilde{C}^*, \mathsf{m}^*_{\mathsf{coin}})$, and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{A}$. Otherwise, it outputs 0.
4. $\mathcal{A}$ outputs coin'. The challenger outputs coin' as the final output of the experiment.

$\mathsf{Hyb}_1^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_0^{\mathsf{coin}}$ except that $\widetilde{C}^*$ is generated as $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{skfe.ct}^*], \mathsf{lock}, 0)$, where $\mathsf{skfe.ct}^* \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Enc}(\mathsf{skfe.msk}, x^*\|0^\lambda)$, $\mathsf{lock} \leftarrow \{0,1\}^\lambda$, and $D[\mathsf{skfe.ct}^*](x)$ is a circuit that has $\mathsf{skfe.ct}^*$ hardwired and outputs $\mathsf{SKFE\text{-}CR\text{-}SKL.CDec}(x, \mathsf{skfe.ct}^*)$.

We pick lock as a uniformly random string that is completely independent of other variables such as $\mathsf{skfe.ct}^*$. Thus, from the security of CC.Obf, we have $\left|\Pr\left[\mathsf{Hyb}_0^{\mathsf{coin}} = 1\right] - \Pr\left[\mathsf{Hyb}_1^{\mathsf{coin}} = 1\right]\right| = \mathsf{negl}(\lambda)$.

$\mathsf{Hyb}_2^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_1^{\mathsf{coin}}$ except that $\mathsf{skfe.ct}^*$ hardwired into the circuit D is generated as $\mathsf{skfe.ct}^* \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Enc}(\mathsf{skfe.msk}, x^*\|\mathsf{lock})$.

From the selective single-ciphertext security of SKFE-CR-SKL, we can show that $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$. Suppose that $\mathsf{Hyb}_1^{\mathsf{coin}} \not\approx \mathsf{Hyb}_2^{\mathsf{coin}}$ and $\mathcal{D}$ is a corresponding distinguisher. We consider the following reduction $\mathcal{R}$:

Execution of $\mathcal{R}^{\mathcal{D}}$ in $\mathsf{Exp}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{R}}(1^\lambda, b)$:

1. $\mathcal{CH}$ runs $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{R}$ with the security parameter 1^λ.
2. $\mathcal{D}$ declares the challenge ciphertext-attribute x^*. $\mathcal{R}$ generates $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and sends $\mathsf{ek} := \mathsf{abe.pk}$ to $\mathcal{D}$.
3. $\mathcal{R}$ chooses $\mathsf{lock} \leftarrow \{0,1\}^\lambda$ and sends $(\mathsf{skfe.m}_0^*, \mathsf{skfe.m}_1^*) := (x^*\|0^\lambda, x^*\|\mathsf{lock})$ to $\mathcal{CH}$.
4. $\mathcal{R}$ simulates the access to $O_{\mathcal{KG}}(y)$ for $\mathcal{D}$ as follows, where $L_{\mathcal{KG}}$ is a list initialized to be empty:

   $O_{\mathcal{KG}}(y)$: Given y, it finds an entry of the form (y, vk_y, V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it generates (sk_y, vk_y) similar to $\mathsf{Hyb}_0^{\mathsf{coin}}$ except that the values $(\mathsf{skfe}.sk_y, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y)$ are generated as $(\mathsf{skfe}.sk_y, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y) \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL}.O_{\mathcal{KG}}(y)$ instead. It adds $(y, \mathsf{vk}_y, \bot)$ to $L_{\mathcal{KG}}$ and returns sk_y.
5. $\mathcal{R}$ simulates the access to $O_{\mathcal{V}rfy}(y, \widetilde{sk})$ as follows:

   $O_{\mathcal{V}rfy}(y, \widetilde{sk})$: Given $(y, \widetilde{sk})$, it finds an entry (y, vk_y, V) from $L_{\mathcal{KG}}$ (If there is no such entry, it returns $\bot$). It then executes a procedure similar to that in $\mathsf{Hyb}_0^{\mathsf{coin}}$, except that $\mathsf{SKFE\text{-}CR\text{-}SKL}.O_{\mathcal{V}rfy}(y, \mathsf{skfe}.sk')$ is executed instead of $\mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{skfe.vk}_y, \mathsf{skfe}.sk')$. The corresponding output d is returned as the output of the oracle.
6. $\mathcal{R}$ requests the challenge ciphertext from $\mathcal{CH}$ and receives $\mathsf{skfe.ct}^* \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Enc}(\mathsf{skfe.msk}, \mathsf{skfe.m}_b^*)$.

7. $\mathcal{D}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{R}$. $\mathcal{R}$ checks that for every entry (y, vk_y, V) such that $R(x^*, y) = 0$, it holds that $V = \top$. If so, it generates $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{skfe.ct}^*], \mathsf{lock}, 0)$ and $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x^* \| \widetilde{C}^*, \mathsf{m}^*_{\mathsf{coin}})$ and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{D}$. Else, it outputs 0.
8. $\mathcal{D}$ outputs a bit b'. $\mathcal{R}$ outputs b' and $\mathcal{CH}$ outputs b' as the final output of the experiment.

It is easy to see that the view of $\mathcal{D}$ is the same as that in $\mathsf{Hyb}_2^{\mathsf{coin}}$ when lock is encrypted in $\mathsf{skfe.ct}^*$ and that of $\mathsf{Hyb}_1^{\mathsf{coin}}$ when 0^λ is encrypted. Moreover, for $\mathcal{D}$ to distinguish between the two hybrids, it must be the case that $V = \top$ for all entries (y, vk_y, V) such that $R(x^*, y) = 0$, which directly implies that the analogous values checked by $\mathsf{SKFE\text{-}CR\text{-}SKL}.O_{\mathcal{V}rfy}$ must also be $\top$. If $R(x^*, y) = 1$, $F(x^* \| 0^\lambda, y) = F(x^* \| \mathsf{lock}, y) = \bot$. Consequently, $\mathcal{R}$ breaks the selective single-ciphertext security of SKFE-CR-SKL. Therefore, it must be that $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$.

$\mathsf{Hyb}_3^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_2^{\mathsf{coin}}$ except that $O_{\mathcal{KG}}$ behaves as follows. (The difference is red colored.)

$O_{\mathcal{KG}}(y)$**:** Given y, it finds an entry of the form (y, vk_y, V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it generates $s\mathcal{K}_y, \mathsf{vk}_y$ as follows.

- Generate $(\mathsf{skfe}.s\mathcal{K}_y, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y) \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{skfe.msk}, y)$. We denote the register holding $\mathsf{skfe}.s\mathcal{K}_y$ as $\mathsf{SKFE.SK}_y$.
- Prepare a register $\mathsf{ABE.R}'_y$ that is initialized to $|0\rangle_{\mathsf{ABE.R}'_y}$.
- To $\mathsf{SKFE.SK}_y$ and $\mathsf{ABE.R}'_y$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}_y} |\beta\rangle_{\mathsf{ABE.R}'_y} \to |u\rangle_{\mathsf{SKFE.SK}_y} \left|\beta \oplus R'(x^* \| \widetilde{C}^*, y \| u)\right\rangle_{\mathsf{ABE.R}'_y}.$$

- Measure $\mathsf{ABE.R}'_y$ in the computational basis and output $\bot$ if the result is 0. Otherwise, go to the next step.
- Prepare a register $\mathsf{ABE.SK}_y$ that is initialized to $|0 \cdots 0\rangle_{\mathsf{ABE.SK}_y}$.
- Sample explicit randomness $\mathsf{k}_y \leftarrow \{0,1\}^\lambda$.
- To $\mathsf{SKFE.SK}_y$ and $\mathsf{ABE.SK}_y$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}_y} |v\rangle_{\mathsf{ABE.SK}_y} \to |u\rangle_{\mathsf{SKFE.SK}_y} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y \| u, \mathsf{k}_y)\rangle_{\mathsf{ABE.SK}_y}$$

  and obtain $s\mathcal{K}_y$ over the registers $\mathsf{SKFE.SK}_y$ and $\mathsf{ABE.SK}_y$.
- Set $\mathsf{vk}_y := (y, \mathsf{abe.msk}, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y, \mathsf{k}_y)$.

It returns $s\mathcal{K}_y$ to $\mathcal{A}$ and adds the entry $(y, \mathsf{vk}_y, \bot)$ to $L_{\mathcal{KG}}$.

With overwhelming probability, we observe that the added procedure that checks $R'(x^* \| \widetilde{C}^*, y \| u)$ in superposition affects the final state $s\mathcal{K}_y$ at most negligibly. We consider the following two cases.

- The first case is where $R(x^*, y) = 1$. In this case, we have $R'(x^* \| \widetilde{C}^*, y \| u) = 1$ for any u. Hence, we see that the added procedure does not affect the state $\mathsf{skfe}.s\mathcal{K}_y$.
- The second case is where $R(x^*, y) = 0$. Let $\mathsf{skfe}.s\mathcal{K}_y = \sum_u \alpha_u |u\rangle_{\mathsf{SKFE.SK}_y}$. From the classical decryption property of SKFE-CR-SKL, we have that for any u, it holds that $\mathsf{SKFE\text{-}CR\text{-}SKL.CDec}(u, \mathsf{skfe.ct}^*) = \mathsf{lock}$. Thus, $\widetilde{C}^*(u) \neq \bot$ and $R'(x^* \| \widetilde{C}^*, y \| u) = 1$ with overwhelming probability from the correctness of CC.Obf. Hence, we see that the added procedure affects the state $\mathsf{skfe.sk}_y$ at most negligibly.

Therefore, we have $\left|\Pr\left[\mathsf{Hyb}_2^{\mathsf{coin}} = 1\right] - \Pr\left[\mathsf{Hyb}_3^{\mathsf{coin}} = 1\right]\right| = \mathsf{negl}(\lambda)$.

$\mathsf{Hyb}_4^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_3^{\mathsf{coin}}$ except that the oracle $O_{\mathcal{V}rfy}$ behaves as follows. (The difference is red colored.)

$O_{\mathcal{V}rfy}(y, \widetilde{sk})$: It finds an entry (y, vk_y, V) from $L_{\mathcal{KG}}$. (If there is no such entry, it returns $\perp$.) It parses $\mathsf{vk}_y = (y, \mathsf{abe.msk}, \mathsf{skfe.vk}_y, \mathsf{skfe.tk}_y, \mathsf{k}_y)$. If $R(x^*, y) = 1$, then it behaves exactly as in $\mathsf{Hyb}_3^{\mathsf{coin}}$ and otherwise, it computes d as follows:

(a) Let the register holding $\widetilde{sk}$ be $\mathsf{SKFE.SK}_y \otimes \mathsf{ABE.SK}_y$.

(b) Prepare a register $\mathsf{SKFE.KT}_y$ that is initialized to $|0\rangle_{\mathsf{SKFE.KT}_y}$.

(c) To $\mathsf{SKFE.SK}_y$ and $\mathsf{SKFE.KT}_y$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}_y} |\beta\rangle_{\mathsf{SKFE.KT}_y} \to |u\rangle_{\mathsf{SKFE.SK}_y} |\beta \oplus \mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{skfe.tk}_y, u)\rangle_{\mathsf{SKFE.KT}_y}.$$

(d) Measure $\mathsf{SKFE.KT}_y$ in the computational basis and set $d := \perp$ if the result is 0. Otherwise, go to the next step.

(e) Prepare a register $\mathsf{ABE.R}'_y$ that is initialized to $|0\rangle_{\mathsf{ABE.R}'_y}$.

(f) To $\mathsf{SKFE.SK}_y$ and $\mathsf{ABE.R}'_y$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}_y} |\beta\rangle_{\mathsf{ABE.R}'_y} \to |u\rangle_{\mathsf{SKFE.SK}_y} \left|\beta \oplus R'(x^* \| \widetilde{C}^*, y \| u)\right\rangle_{\mathsf{ABE.R}'_y}.$$

(g) Measure $\mathsf{ABE.R}'_y$ in the computational basis and set $d := \perp$ if the result is 0. Otherwise, go to the next step.

(h) To $\mathsf{SKFE.SK}_y$ and $\mathsf{ABE.SK}_y$, apply the map

$$|u\rangle_{\mathsf{SKFE.SK}_y} |v\rangle_{\mathsf{ABE.SK}_y} \to |u\rangle_{\mathsf{SKFE.SK}_y} |v \oplus \mathsf{ABE.KG}(\mathsf{abe.msk}, y \| u, \mathsf{k})\rangle_{\mathsf{ABE.SK}_y}.$$

(i) Trace out the register $\mathsf{ABE.SK}_y$ and obtain $\mathsf{skfe}.sk'$ over $\mathsf{SKFE.SK}_y$.

(j) Set $d := \top$ if $\top = \mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{skfe.vk}_y, \mathsf{skfe}.sk')$ and set $d := \perp$ otherwise. It returns d to $\mathcal{A}$. Finally, if $V = \perp$, it updates the entry (y, vk_y, V) into (y, vk_y, d).

Suppose there exists a QPT distinguisher $\mathcal{D}$ that has non-negligible advantage in distinguishing $\mathsf{Hyb}_3^{\mathsf{coin}}$ and $\mathsf{Hyb}_4^{\mathsf{coin}}$. Let $\mathcal{D}$ make $q = \mathrm{poly}(\lambda)$ many queries to the oracle $O_{\mathcal{V}rfy}(\cdot,\cdot)$. We will now consider the following QPT algorithm $\mathcal{A}_{\mathsf{o2h}}$ with access to an oracle $\widetilde{O_{\mathcal{KG}}}$ and an oracle $\mathcal{O}$ that runs $\mathcal{D}$ as follows:

$\underline{\mathcal{A}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}}, \mathcal{O}}(\mathsf{abe.pk}, \mathsf{skfe.msk})}$:

1. $\mathcal{A}_{\mathsf{o2h}}$ runs $\mathcal{D}$ who sends the challenge ciphertext-attribute x^* to $\mathcal{A}_{\mathsf{o2h}}$.
2. $\mathcal{A}_{\mathsf{o2h}}$ sends $\mathsf{ek} = \mathsf{abe.pk}$ to $\mathcal{D}$ and initializes $L_{\mathcal{KG}}$ to be an empty list.
3. When $\mathcal{D}$ queries $O_{\mathcal{KG}}$ on input y, $\mathcal{A}_{\mathsf{o2h}}$ queries the oracle $\widetilde{O_{\mathcal{KG}}}(y)$ in order to obtain its response $|\phi\rangle$ and a list $\widetilde{L_{\mathcal{KG}}}$. It updates $L_{\mathcal{KG}} = \widetilde{L_{\mathcal{KG}}}$. It sends $|\phi\rangle$ to $\mathcal{D}$.
4. $\mathcal{A}_{\mathsf{o2h}}$ simulates the access of $O_{\mathcal{V}rfy}$ for $\mathcal{D}$ as follows:

   $\underline{O_{\mathcal{V}rfy}(y, \widetilde{sk})}$:

   (a) Execute Steps (a)-(e) of $O_{\mathcal{V}rfy}$ as in $\mathsf{Hyb}_4^{\mathsf{coin}}$.

   (b) Apply the map $|u\rangle_{\mathsf{SKE.SK}_y} |\beta\rangle_{\mathsf{ABE.R}_y} \to |u\rangle_{\mathsf{SKE.SK}_y} |\beta \oplus \mathcal{O}(u)\rangle_{\mathsf{ABE.R}_y}$ to the registers $\mathsf{SKE.SK}_y$ and $\mathsf{ABE.R}_y$.

   (c) Execute Steps (g)-(j) of $O_{\mathcal{V}rfy}$ as in $\mathsf{Hyb}_4^{\mathsf{coin}}$.
5. $\mathcal{D}$ sends $(\mathsf{m}_0^*, \mathsf{m}_1^*) \in \mathcal{M}^2$ to $\mathcal{A}_{\mathsf{o2h}}$. $\mathcal{A}_{\mathsf{o2h}}$ checks if for every entry (y, vk_y, V) in $L_{\mathcal{KG}}$ such that $R(x^*, y) = 0$, it holds that $V = \top$. If so, it generates $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{skfe.ct}^*], \mathsf{lock}, 0)$ and $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x^* \| \widetilde{C}^*, \mathsf{m}^*_{\mathsf{coin}})$, where $\mathsf{skfe.ct}^* = \mathsf{SKFE\text{-}CR\text{-}SKL.Enc}(\mathsf{skfe.msk}, x^* \| 0^\lambda)$ and sends $\mathsf{ct}^* := \mathsf{abe.ct}^*$ to $\mathcal{D}$. Otherwise, it outputs 0.

6. $\mathcal{D}$ outputs a guess b'. $\mathcal{A}_{\mathsf{o2h}}$ outputs b'.

Let H be an oracle that for every input u, outputs 1. Consider now the extractor $\mathcal{B}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}},H}$ as specified by the O2H Lemma (Lemma 3.2). We will now construct a reduction $\mathcal{R}$ that runs $\mathcal{B}_{\mathsf{o2h}}$ by simulating the oracles $\widetilde{O_{\mathcal{KG}}}$ and H for $\mathcal{B}_{\mathsf{o2h}}$, and breaks key-testability of the SKFE-CR-SKL scheme.

Execution of $\mathcal{R}$ in $\mathsf{Exp}^{\mathsf{key\text{-}test}}_{\mathsf{SKFE\text{-}CR\text{-}SKL},\mathcal{R}}(1^\lambda)$:

1. The challenger $\mathcal{CH}$ runs $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{R}$ with input $\mathsf{skfe.msk}$.
2. $\mathcal{R}$ samples $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{ABE.Setup}(1^\lambda)$ and initializes $\mathcal{B}_{\mathsf{o2h}}$ with the input $(\mathsf{abe.pk}, \mathsf{skfe.msk})$.
3. $\mathcal{R}$ simulates the access to $\widetilde{O_{\mathcal{KG}}}(y)$ for $\mathcal{B}$ as follows, where $L_{\mathcal{KG}}$ is a list initialized to be empty:

   $\widetilde{O_{\mathcal{KG}}}(y)$: Given y, it finds an entry of the form (y, vk_y, V) from $L_{\mathcal{KG}}$ (If there is such an entry, it returns $(\perp, L_{\mathcal{KG}})$). Otherwise, it generates $(\mathsf{sk}_y, \mathsf{vk}_y)$ similar to $\mathsf{Hyb}_3^{\mathsf{coin}}$. It adds $(y, \mathsf{vk}_y, \perp)$ to $L_{\mathcal{KG}}$ and returns $(\mathsf{sk}_y, L_{\mathcal{KG}})$.
4. When $\mathcal{B}_{\mathsf{o2h}}$ queries an input u to H, $\mathcal{R}$ responds with 1.
5. $\mathcal{B}_{\mathsf{o2h}}$ outputs values y and sk. $\mathcal{R}$ sends $(\mathsf{sk}, y, \mathsf{lock})$ to $\mathcal{CH}$.

We will now claim that with non-negligible probability, $\mathcal{R}$ obtains values sk and y such that $R'(x^*\|\widetilde{C}^*, y\|\mathsf{sk}) = 0$. Recall that the hybrids differ only when $R(x^*, y) = 0$. By the definition of R' and $\widetilde{C}^*$ and the decryption correctness of SKFE-CR-SKL, this will imply that $\mathsf{SKFE\text{-}CR\text{-}SKL.CDec}(\mathsf{sk}, \mathsf{skfe.ct}^\star) \neq F(x^*\|\mathsf{lock}, y)$ since $F(x^*\|\mathsf{lock}, y) = \mathsf{lock}$, and $R'(x^*\|\widetilde{C}^*, y\|\mathsf{sk}) = 0$ requires $\widetilde{C}^*(\mathsf{sk}) = \perp$. Moreover, $\mathsf{KeyTest}(\mathsf{skfe.tk}_y, \mathsf{sk})$ also holds. Consequently, this will imply $\mathcal{R}$ breaks the key-testability of SKFE-CR-SKL. To prove this, we will rely on the One-Way to Hiding (O2H) Lemma (Lemma 3.2). Consider an oracle G which takes as input u and outputs $R'(x^*\|\widetilde{C}^\star, y\|u)$ and an oracle H which takes as input u and outputs 1. Notice that if the oracle $\mathcal{O} = G$, then the view of $\mathcal{D}$ as run by $\mathcal{A}_{\mathsf{o2h}}$ is the same as in $\mathsf{Hyb}_4^{\mathsf{coin}}$, while if $\mathcal{O} = H$, the view of $\mathcal{D}$ is the same as in $\mathsf{Hyb}_3^{\mathsf{coin}}$. By the O2H Lemma, we have the following, where $z = (\mathsf{abe.pk}, \mathsf{skfe.msk})$.

$$\left|\Pr\left[\mathcal{A}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}},H}(z) = 1\right] - \Pr\left[\mathcal{A}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}},G}(z) = 1\right]\right| \leq 2q \cdot \sqrt{\Pr\left[\mathcal{B}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}},H}(z) \in S\right]} \ .$$

where S is a set where the oracles H and G differ, which happens only for inputs u s.t. $R'(x^*\|\widetilde{C}^\star, y\|u) = 0$. Since $\mathcal{R}$ obtains sk and y as the output of $\mathcal{B}_{\mathsf{o2h}}^{\widetilde{O_{\mathcal{KG}}},H}(z)$, the argument follows that $\mathsf{Hyb}_3^{\mathsf{coin}} \approx \mathsf{Hyb}_4^{\mathsf{coin}}$.

$\mathsf{Hyb}_5^{\mathsf{coin}}$**:** This is the same as $\mathsf{Hyb}_4^{\mathsf{coin}}$ except that $\mathsf{ct}^* := \mathsf{abe.ct}^*$ is generated as $\mathsf{abe.ct}^* \leftarrow \mathsf{ABE.Enc}(\mathsf{abe.pk}, x^*\|\widetilde{C}^*, 0^{\ell_{\mathsf{m}}})$.

The view of $\mathcal{A}$ in $\mathsf{Hyb}_4^{\mathsf{coin}}$ and $\mathsf{Hyb}_5^{\mathsf{coin}}$ can be simulated with $\mathsf{abe.pk}$ and the access to the quantum key generation oracle $O_{\mathtt{qkg}}$. This is because before ABE.KG is required to be applied in the simulation of oracles $O_{\mathcal{KG}}$ and $O_{\mathcal{V}\mathit{rfy}}$, the relation check $R'(x^*\|\widetilde{C}^\star, y\|u)$ is already applied in superposition. Thus, we have $\left|\Pr\left[\mathsf{Hyb}_4^{\mathsf{coin}} = 1\right] - \Pr\left[\mathsf{Hyb}_5^{\mathsf{coin}} = 1\right]\right| = \mathsf{negl}(\lambda)$.

Lastly, Hyb_5^0 and Hyb_5^1 are exactly the same experiment and thus we have $\left|\Pr\left[\mathsf{Hyb}_5^0 = 1\right] - \Pr\left[\mathsf{Hyb}_5^1 = 1\right]\right| = \mathsf{negl}(\lambda)$. Then, from the above arguments, we obtain

$$\begin{aligned}&\left|\Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 0) = 1\right] - \mathsf{Exp}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, 1) = 1]\right| \\ =&\left|\Pr\left[\mathsf{Hyb}_0^0 = 1\right] - \Pr\left[\mathsf{Hyb}_0^1 = 1\right]\right| \leq \mathsf{negl}(\lambda).\end{aligned}$$

This completes the proof. □

Recall now that SKFE-CR-SKL with Key-Testability is implied by BB84-based SKE-CD, OWFs and adaptively single-ciphertext function-private SKFE. Since this notion of SKFE is implied by OWFs (Theorem 7.3), we have that SKFE-CR-SKL with Key-Testability is implied by LWE. Since, Compute-and-Compare Obfuscation and quantum-secure Ciphertext-Policy ABE for Circuits are both implied by LWE, this gives us the following theorem:

Theorem 7.13. *There exists an ABE-CR-SKL scheme satisfying selective IND-KLA security, assuming the polynomial hardness of the LWE assumption.*

8 ABE-CR^2-SKL from Multi-Input ABE

In this section, we show how to achieve ABE-CR-SKL with classical certificates (ABE-CR^2-SKL) from MI-ABE.

8.1 Definitions of Multi-Input ABE

First, we recall the definitions of MI-ABE. The syntax of MI-ABE is as follows:

Definition 8.1. *A Multi-Input (Ciphertext-Policy) Attribute-Based Encryption scheme* MI-ABE *is a tuple of four PPT algorithms* $(\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$. *Let* $k = \mathrm{poly}(\lambda)$ *and let* $\{\mathcal{X}_\lambda\}_\lambda$, $\{(\mathcal{Y}_\lambda)^k\}_\lambda$ *and* $R = \{R_\lambda : \mathcal{X}_\lambda \times (\mathcal{Y}_\lambda)^k \to \{0,1\}\}$ *be the ciphertext attribute space, key attribute space and the relation associated with* MI-ABE *respectively. Let* $\{0,1\}^\ell$ *be the message space. Let* $s(\lambda)$ *denote the maximum bit string length required to describe PPT circuits with input size* $k \cdot n(\lambda)$ *and depth* $d(\lambda)$ *for polynomials* s, n *and* d. *Let* $\mathcal{X}_\lambda = \{0,1\}^{s(\lambda)}$ *and* $\mathcal{Y} = \{0,1\}^{n(\lambda)}$. *Let* R *be such that* $R(x, y_1, \ldots, y_k) = 0 \iff x(y_1, \ldots, y_k) = \bot$ *where* $x \in \mathcal{X}$ *is parsed as a* k*-input circuit and* $(y_1, \ldots, y_k) \in \mathcal{Y}^k$.

$\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter* 1^λ *and outputs a public key* pk *and master secret key* msk.

$\mathsf{KG}(\mathsf{msk}, i, y_i) \to \mathsf{sk}_i$**:** *The key generation algorithm takes the master secret key* msk, *an index* $i \in [k]$, *and a key-attribute* $y_i \in \mathcal{Y}$ *as input. It outputs a secret-key* sk_i.

$\mathsf{Enc}(\mathsf{pk}, x, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes the public key* pk, *a ciphertext attribute* $x \in \mathcal{X}$, *and a message* $\mathsf{m} \in \{0,1\}^\ell$ *as input. It outputs a ciphertext* ct.

$\mathsf{Dec}(\mathsf{ct}, \mathsf{sk}_1, \ldots \mathsf{sk}_k) \to \widetilde{\mathsf{m}}$**:** *The decryption algorithm takes as input a ciphertext* ct *and* k *secret-keys* $\mathsf{sk}_1, \ldots, \mathsf{sk}_k$. *It outputs a value* $\widetilde{\mathsf{m}} \in \{0,1\}^\ell \cup \bot$.

Correctness: *We require that*

$$\Pr\left[\mathsf{Dec}(\mathsf{ct}, \mathsf{sk}_1, \ldots, \mathsf{sk}_k) = \mathsf{m} \;\middle|\; \begin{array}{l} (\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda), \\ \forall i \in [k] : \mathsf{sk}_i \leftarrow \mathsf{KG}(\mathsf{msk}, i, y_i), \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{pk}, x, \mathsf{m}) \end{array}\right] \geq 1 - \mathsf{negl}(\lambda)$$

holds for all $x \in \mathcal{X}$ *and* $(y_1, \ldots, y_k) \in \mathcal{Y}^k$ *such that* $R(x, y_1, \ldots, y_k) = 0$ *and* $\mathsf{m} \in \{0,1\}^\ell$.

Remark 8.2. We say that an MI-ABE scheme has *polynomial-arity*, if it allows k to be an arbitrary polynomial in λ.

Definition 8.3 (Post-Quantum Selective Security for MI-ABE:). *We say that* MI-ABE *is a selective-secure MI-ABE scheme for relation* $R : \mathcal{X} \times \mathcal{Y}^k \to \{0,1\}$, *if it satisfies the following requirement, formalized from the experiment* $\mathsf{Exp}^{\mathsf{sel\text{-}ind}}_{\mathsf{MI\text{-}ABE},\mathcal{A}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger* Ch*:*

1. $\mathcal{A}$ *declares the challenge ciphertext attribute* x. Ch *runs* $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ *and sends* pk *to* $\mathcal{A}$.
2. $\mathcal{A}$ *can get access to the following key generation oracle.*

   $O_{\mathtt{kg}}(i, y_i)$**:** *It outputs* $\mathsf{sk}_i \leftarrow \mathsf{KG}(\mathsf{msk}, i, y_i)$ *to* $\mathcal{A}$.

3. *At some point,* $\mathcal{A}$ *sends* $(\mathsf{m}_0, \mathsf{m}_1)$ *to* Ch. *Then,* Ch *generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{pk}, x, \mathsf{m}_{\mathsf{coin}})$ *and sends* ct^* *to* $\mathcal{A}$.

4. *Again,* $\mathcal{A}$ *can get access to the oracle* O_{kg}.

5. $\mathcal{A}$ *outputs a guess* coin' *for* coin *and the experiment outputs* coin'.

For an adversary to be admissible, we require that for all tuples $(y_1, \ldots, y_k) \in \mathcal{Y}^k$ *received by* $\mathcal{A}$*, it must hold that* $R(x, y_1, \ldots, y_k) = 1$ *(non-decrytable). Given this constraint, we say* MI-ABE *satisfies selective security if for all QPT* $\mathcal{A}$*, the following holds:*

$$\mathsf{Adv}^{\mathsf{sel\text{-}ind}}_{\mathsf{MI\text{-}ABE},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}ind}}_{\mathsf{MI\text{-}ABE},\mathcal{A}}(1^\lambda, 0) \to 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{sel\text{-}ind}}_{\mathsf{MI\text{-}ABE},\mathcal{A}}(1^\lambda, 1) \to 1\right]\right| \leq \mathsf{negl}(\lambda).$$

Remark 8.4. We do not require quantum security in Definition 8.3 unlike Definition 3.5.

Comparison with the Definition of Agrawal et. al [ARYY23]. Their definition considers a key-policy variant of MI-ABE. It consists of algorithms $(\mathsf{Setup}, \mathsf{Enc}, \mathsf{KG}_1, \cdots, \mathsf{KG}_{k-1}, \mathsf{KG}_k, \mathsf{Dec})$ where Enc works in a similar way as our definition, i.e., $\mathsf{Enc}(\mathsf{pk}, x_0, \mathsf{m}) \to \mathsf{ct}$ for message m and attribute x_0. For each $i \in [k-1]$, KG_i works as $\mathsf{KG}_i(\mathsf{msk}, x_i) \to \mathsf{sk}_i$ for the i-th key-attribute x_i. On the other hand, KG_k works as $\mathsf{KG}_k(\mathsf{msk}, f) \to \mathsf{sk}_f$ where f is an arbitrary k-input function. The guarantee is that decryption is feasible if and only if $f(x_0, \ldots, x_{k-1}) = \bot$. The algorithm Dec has the syntax $\mathsf{Dec}(\mathsf{pk}, \mathsf{ct}, \mathsf{sk}_1, \ldots, \mathsf{sk}_{k-1}, \mathsf{sk}_f) \to \mathsf{m}'$.

It is easy to see that this definition directly implies a $(k-1)$-input ciphertext-policy MI-ABE with algorithms $(\mathsf{Setup}', \mathsf{Enc}', \mathsf{KG}', \mathsf{Dec}')$. Specifically, $\mathsf{Setup}'(1^\lambda) := \mathsf{Setup}(1^\lambda)$, $\mathsf{Enc}'(\mathsf{pk}, x_0, \mathsf{m}) := \big(\mathsf{pk}, \mathsf{Enc}(\mathsf{pk}, x_0, \mathsf{m})\big)$ and KG' can be defined as $\mathsf{KG}'(\mathsf{msk}, i, x_i) := \mathsf{KG}_i(\mathsf{msk}, x_i)$ for all $i \in [k-1]$. Finally, Dec' is defined as $\mathsf{Dec}'(\mathsf{ct} = (\mathsf{pk}, \widetilde{\mathsf{ct}}), \mathsf{sk}_1, \ldots, \mathsf{sk}_{k-1}) := \mathsf{Dec}(\mathsf{pk}, \widetilde{\mathsf{ct}}, \mathsf{sk}_1, \ldots, \mathsf{sk}_{k-1}, \mathsf{sk}_f)$ for the function $f(x_0, \ldots, x_{k-1}) = C_{x_0}(x_1, \ldots, x_{k-1})$ where C_{x_0} is the circuit described by ciphertext-attribute x_0.

Unfortunately, [ARYY23] only allows for a constant k, and hence we do not currently know of a construction where $k = \mathrm{poly}(\lambda)$. In the recent work of [AKY24], a variant of MI-ABE (for $k = \mathrm{poly}(\lambda)$) was achieved from the LWE and evasive LWE assumptions, which is non-standard. However, the scheme it implies would require the encryption algorithm Enc' to utilize the master secret-key msk.

Disucssion on MI-ABE and IO. We focus on MI-ABE satisfying Definition 8.1 in this paragraph. MI-ABE is not stronger than IO since IO is equivalent to multi-input functional encryption (MIFE) [GGG+14] and MIFE trivially implies MI-ABE. Although we do not know whether MIFE is separated from MI-ABE, it is likely since PKFE is separated from ABE [GMM17] and constructing FE schemes is significantly more challenging than ABE schemes. Moreover, MI-ABE is not currently known from weaker assumptions than IO. They are qualitatively different primitives, and MI-ABE could be constructed from weaker assumptions in the future. We also know that MI-ABE implies witness encryption [BJK+18], and witness encryption is achieved from the evasive LWE assumption [Tsa22, VWW22]. Hence, MI-ABE is somewhere between witness encryption and IO. Achieving MI-ABE from lattice assumptions is an interesting open problem.

8.2 Definitions of ABE-CR^2-SKL

The syntax of ABE-CR^2-SKL is defined as follows.

Definition 8.5 (ABE-CR^2-SKL). *An ABE-CR^2-SKL scheme* $\mathsf{ABE\text{-}CR^2\text{-}SKL}$ *is a tuple of six algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Del}, \mathsf{Vrfy})$. *Below, let* $\mathcal{M}$ *be the message space of* $\mathsf{ABE\text{-}CR^2\text{-}SKL}$*,* $\mathcal{X}$ *be the ciphertext-attribute space, and* $\mathcal{Y}$ *the key-attribute space.*

$\mathsf{Setup}(1^\lambda) \to (\mathsf{ek}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter* 1^λ*, and outputs an encryption key* ek *and a master secret-key* msk.

$\mathcal{KG}(\mathsf{msk}, y) \to (sk, \mathsf{vk})$**:** *The key generation algorithm takes the master secret-key* msk *and a key-attribute* $y \in \mathcal{Y}$ *as inputs, and outputs a decryption key* sk *and a verification key* vk*.*

$\mathsf{Enc}(\mathsf{ek}, \widetilde{x}, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes an encryption key* ek*, a ciphertext-attribute* $\widetilde{x} \in \mathcal{X}$*, and a message* $\mathsf{m} \in \mathcal{M}$ *as inputs, and outputs a ciphertext* ct*.*

$\mathcal{Dec}(sk, \mathsf{ct}) \to \widetilde{\mathsf{m}}$**:** *The decryption algorithm takes a decryption key* sk *and a ciphertext* ct*, and outputs a value* $\widetilde{\mathsf{m}}$ *or* $\perp$*.*

$\mathcal{Del}(sk) \to \mathsf{cert}$**:** *The deletion algorithm takes a decryption key* sk *and outputs a deletion certificate* cert*.*

$\mathsf{Vrfy}(\mathsf{vk}, \mathsf{cert}') \to \top / \perp$**:** *The verification algorithm takes a verification key* vk *and a certificate* cert'*, and outputs* $\top$ *or* $\perp$*.*

Decryption correctness: *For every* $\mathsf{m} \in \mathcal{M}$*,* $\widetilde{x} \in \mathcal{X}$ *and* $y \in \mathcal{Y}$ *such that* $\widetilde{x}(y) = 0$ *(decryptable), we have*

$$\Pr\left[\mathcal{Dec}(sk, \mathsf{ct}) = \mathsf{m} \;:\; \begin{array}{l}(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (sk, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y) \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{ek}, \widetilde{x}, \mathsf{m})\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *For every* $y \in \mathcal{Y}$*, we have*

$$\Pr\left[\mathsf{Vrfy}(\mathsf{vk}, \mathsf{cert}) = \top \;:\; \begin{array}{l}(\mathsf{ek}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (sk, \mathsf{vk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y) \\ \mathsf{cert} \leftarrow \mathcal{Del}(sk)\end{array}\right] \geq 1 - \mathsf{negl}(\lambda).$$

Remark 8.6. We use the same experiment identifier $\mathsf{sel\text{-}ind\text{-}kla}$ to refer to the selective IND-KLA security experiments of ABE-CR-SKL and ABE-CR^2-SKL. This is for the sake of simplicity, as the exact experiment will be clear from the context.

Selective IND-KLA Security: *This security notion for an ABE-CR^2-SKL scheme is defined in the same way as for ABE-CR-SKL, except that instead of access to the oracle* $O_{\mathcal{V}rfy}$ *in the experiment,* $\mathcal{A}$ *receives access to the following oracle:*

$O_{\mathsf{Vrfy}}(y, \mathsf{cert})$**:** *Given* (y, cert)*, it finds an entry* (y, vk, V) *from* $L_{\mathcal{KG}}$*. (If there is no such entry, it returns* $\perp$*.) It then runs* $d := \mathsf{Vrfy}(\mathsf{vk}, \mathsf{cert})$ *and returns* d *to* $\mathcal{A}$*. If* $V = \perp$*, it updates the entry into* (y, vk, d)*.*

8.3 Construction

Construction overview. We first provide the main idea behind our ABE-CR^2-SKL based on MI-ABE, which follows along the lines of our ABE-CR-SKL construction with some key-differences. In this case, we directly rely on a BB84-based SKE-CD scheme as a building block, instead of needing something similar to SKFE-CR-SKL. Consider now an SKE-CD ciphertext $\mathsf{skecd}.ct$ of the plaintext 0^λ. Let its corresponding verification-key $\mathsf{skecd.vk}$ be of the form $\mathsf{skecd.vk} = (x, \theta)$, where x and θ are k-bit strings. Then, $\mathsf{skecd}.ct$ is of the form $|\psi_1\rangle \otimes \cdots \otimes |\psi_k\rangle$, where for $i \in [k]$, $|\psi_i\rangle$ is of the following form:

$$|\psi_i\rangle = \begin{cases} |x[i]\rangle & if\ \ \theta[i] = 0 \\ |0\rangle + (-1)^{x[i]}\,|1\rangle & if\ \ \theta[i] = 1 \end{cases}$$

Now, for each $i \in \{2, \ldots, k\}$ and $u \in \{0, 1\}$, consider the attribute-key $\mathsf{abe.sk}_{i,u}$ generated by the MI-ABE key-generation algorithm for slot i and attribute $t \| u$. Here, t is a value chosen at random and is common for all slots corresponding to a given decryption-key sk. Additionally, for $u \in \{0, 1\}$, consider the keys $\mathsf{abe.sk}_{1,u}$ corresponding to the attributes $t \| u \| y$ where y is the actual ABE key-attribute.

Consider now the following state ρ_i for each $i \in [k]$:

$$\rho_i = \begin{cases} |x[i]\rangle \left|\mathsf{abe.sk}_{i,x[i]}\right\rangle & if\ \ \theta[i]=0 \\ |0\rangle\,|\mathsf{abe.sk}_{i,0}\rangle + (-1)^{x[i]}\,|1\rangle\,|\mathsf{abe.sk}_{i,1}\rangle & if\ \ \theta[i]=1, \end{cases}$$

The quantum decryption-key of our scheme will be the tuple $s\mathcal{k} = (\rho_i)_{i\in[k]}$. The encryption algorithm is similar to the ABE-CR-SKL scheme, and outputs an MI-ABE encryption of message m under a policy $C_{\mathsf{ABE}}\|\widetilde{C}$ where $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda)$ is a simulator of a compute and compare obfuscator and C_{ABE} is the actual ABE policy. However, the MI-ABE relation is a little different. The relation is such that for a set of k attributes $y_1 = t_1\|u_1\|y$ and $y_2 = t_2\|u_2, \ldots, y_k = t_k\|u_k$, it outputs 0 (decryptable) whenever $C_{\mathsf{ABE}}(y) = 0$ (the ABE relation is satisfied) AND $t_1 = \ldots = t_k$ AND $\widetilde{C}(u_1\| \ldots \|u_k) = \perp$. Otherwise, it outputs 1. As in ABE-CR-SKL, the idea is that $\widetilde{C}$ is indistinguishable from $\widetilde{C}^*$ that is an obfuscation of the compute-and-compare circuit $\mathbf{CC}[D[\mathsf{lock}\oplus r, \mathsf{skecd.sk}], \mathsf{lock}, 0]$. Here, lock and r are random values, and $\mathsf{skecd.sk}$ is the secret-key of the SKE-CD scheme. The circuit D is such that $D(u)$ outputs $\mathsf{lock}\oplus r\oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u)$, where $\mathsf{SKECD.CDec}$ is the classical decryption algorithm of the SKE-CD scheme.

Consequently, when every $s\mathcal{k} = (\rho_i)_{i\in[k]}$ the adversary receives is generated using an SKE-CD encryption of r (instead of 0^λ as in the scheme), then any tuple of k MI-ABE keys of the adversary having attributes $\left(t_1\|u_1\|y, t_2\|u_2 \ldots, t_k\|u_k\right)$ satisfies one of the three conditions:

- The values $t_1 \ldots t_k$ are not all the same.
- $C_{\mathsf{ABE}}(y) \neq 0$.
- For $u = u_1\| \ldots \|u_k$, $D[\mathsf{lock}\oplus r, \mathsf{skecd.sk}](u)$ returns lock.

Notice that the former condition ensures that the adversary cannot interleave keys corresponding to different decryption-keys. The last condition holds because the adversary never receives $\mathsf{abe.sk}_{i,1-x[i]}$ for any i such that $\theta[i] = 0$. Consequently, u and x are the same at all positions where $\theta[i] = 0$. This means that $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u)$ outputs r by the classical decryption property of SKECD (Definition 3.18). It is important to note that the positions i where $\theta[i] = 1$ (the Hadamard positions) have no effect on the value output by $\mathsf{SKECD.CDec}$ as their purpose is just in the verification of deletion. As a result, the security of MI-ABE allows to simulate the adversary's view in this hybrid, as no "decryptable" set of k keys is given out.

Importantly, the switch from SKE-CD encryptions of 0^λ to r is indistinguishable, given that the adversary deletes all the information in the SKE-CD ciphertexts. To enforce this, the deletion algorithm requires the adversary to measure both the SKE-CD and MI-ABE registers for each slot to obtain values $(c_i, d_i)_{i\in[k]}$. Then, given the values $\{\mathsf{abe.sk}_{i,0}\oplus \mathsf{abe.sk}_{i,1}\}_{i\in[k]}$ as part of the verification key, the verification checks whether $x[i] = c_i \oplus d_i \cdot (\mathsf{abe.sk}_{i,0}\oplus \mathsf{abe.sk}_{i,1})$ holds for every $i \in [k]$ such that $\theta[i] = 1$. As a result, we are able use a standard hybrid argument to turn any distinguisher (of the 0^λ and r hybrids) into an attack on the certified deletion security of the SKE-CD scheme.

Construction. We will construct an ABE-CR^2-SKL scheme $\mathsf{ABE\text{-}CR^2\text{-}SKL} = \mathsf{ABE\text{-}CR^2\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{D}ec, \mathcal{D}el, \mathsf{Vrfy})$ using the following building blocks:

- Multi-Input (Ciphertext-Policy) ABE Scheme $\mathsf{MI\text{-}ABE} = \mathsf{MI\text{-}ABE}.(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$ for the following relation:

  $R(\widetilde{x}\|z, y_1, \cdots, y_k)$**:** Let $\widetilde{x}$ be interpreted as a circuit and z as a k-input circuit.
  - Parse $y_1 = t_1\|u_1\|y$.
  - Parse $y_i = t_i\|u_i$ for every $i \in 2, \ldots, k$.
  - If $t_i \neq t_j$ for some $i, j \in [k]$, output 1. Otherwise, go to the next step.
  - If $z(u_1\| \cdots \|u_k) = \perp$ AND $\widetilde{x}(y) = 0$, output 0 (decryptable). Else, output 1.

- Compute-and-Compare Obfuscation $\mathsf{CC.Obf}$ with the simulator $\mathsf{CC.Sim}$.
- BB84-based SKE-CD scheme $\mathsf{SKECD} = \mathsf{SKECD}.(\mathsf{KG}, \mathcal{Enc}, \mathcal{Dec}, \mathcal{Del}, \mathsf{Vrfy})$ with the classical decryption algorithm $\mathsf{SKECD.CDec}$.

The description of each algorithm of $\mathsf{ABE\text{-}CR^2\text{-}SKL}$ is as follows.

$\mathsf{ABE\text{-}CR^2\text{-}SKL.Setup}(1^\lambda)$:

- Generate $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{MI\text{-}ABE.Setup}(1^\lambda)$.
- Generate $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$.
- Output $\mathsf{ek} := \mathsf{abe.pk}$ and $\mathsf{msk} := (\mathsf{abe.msk}, \mathsf{skecd.sk})$.

$\mathsf{ABE\text{-}CR^2\text{-}SKL}.\mathcal{KG}(\mathsf{msk}, y)$:

- Parse $\mathsf{msk} = (\mathsf{abe.msk}, \mathsf{ske.msk})$.
- Sample a random value $t \leftarrow \{0,1\}^\lambda$.
- Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{Enc}(\mathsf{skecd.sk}, 0^\lambda)$. $\mathsf{skecd.vk}$ is of the form $(x, \theta) \in \{0,1\}^k \times \{0,1\}^k$, and $\mathsf{skecd}.ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT_1}} \otimes \cdots \otimes |\psi_k\rangle_{\mathsf{SKECD.CT_k}}$.
- Generate $\mathsf{abe.sk}_{1,b} \leftarrow \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, 1, t\|b\|y)$ for each $b \in \{0,1\}$.
- For every $i \in 2, \ldots, k$, do the following:
  - Generate $\mathsf{abe.sk}_{i,b} \leftarrow \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, i, t\|b)$ for each $b \in \{0,1\}$.
- For every $i \in [k]$, do the following:
  - Prepare a register $\mathsf{ABE.SK_i}$ that is initialized to $|0\cdots0\rangle_{\mathsf{ABE.SK_i}}$.
  - To $\mathsf{SKECD.CT_i}$ and $\mathsf{ABE.SK_i}$, apply the map
$$|u\rangle_{\mathsf{SKECD.CT_i}} |v\rangle_{\mathsf{ABE.SK_i}} \to |u\rangle_{\mathsf{SKECD.CT_i}} |v \oplus \mathsf{abe.sk}_{i,u}\rangle_{\mathsf{ABE.SK_i}}$$
    and obtain the resulting state ρ_i.
- Output $s\mathcal{K} := (\rho_i)_{i\in[k]}$ and $\mathsf{vk} := \left(\mathsf{skecd.vk}, \{\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1}\}_{i\in[k]:\theta[i]=1}\right)$.

$\mathsf{ABE\text{-}CR^2\text{-}SKL.Enc}(\mathsf{ek}, \widetilde{x}, \mathsf{m})$:

- Parse $\mathsf{ek} = \mathsf{abe.pk}$.
- Generate $\widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$, where pp_D consists of circuit parameters of D defined in the security proof.
- Generate $\mathsf{abe.ct} \leftarrow \mathsf{MI\text{-}ABE.Enc}(\mathsf{abe.pk}, \widetilde{x}\|\widetilde{C}, \mathsf{m})$.
- Output $\mathsf{ct} := \mathsf{abe.ct}$.

$\mathsf{ABE\text{-}CR^2\text{-}SKL}.\mathcal{Dec}(s\mathcal{K}, \mathsf{ct})$:

- Parse $s\mathcal{K} = (\rho_i)_{i\in[k]}$ and $\mathsf{ct} = \mathsf{abe.ct}$. We denote the register holding ρ_i as $\mathsf{SKECD.CT_i} \otimes \mathsf{ABE.SK_i}$.
- Prepare a register MSG that is initialized to $|0\cdots0\rangle_{\mathsf{MSG}}$
- To registers $\bigotimes_{i\in[k]} \mathsf{ABE.SK_i}$ and MSG, apply the map $\bigotimes_{i\in[k]} |v_i\rangle_{\mathsf{ABE.SK_i}} \otimes |w\rangle_{\mathsf{MSG}} \to \bigotimes_{i\in[k]} |v_i\rangle_{\mathsf{ABE.SK_i}} \otimes |w \oplus \mathsf{MI\text{-}ABE.Dec}(\mathsf{abe.ct}, v_1, \cdots, v_k)\rangle_{\mathsf{MSG}}$.
- Measure the register MSG in the computational basis and output the result m'.

$\mathsf{ABE\text{-}CR^2\text{-}SKL}.\mathcal{Del}(s\mathcal{K})$:

- Parse $s\mathcal{K} = (\rho_i)_{i\in[k]}$. Let the register holding ρ_i be denoted as $\mathsf{SKECD.CT_i} \otimes \mathsf{ABE.SK_i}$.

- For each $i \in [k]$, measure the registers SKECD_i and $\mathsf{ABE.SK}_i$ in the Hadamard basis to obtain outcomes c_i and d_i.
- Output $\mathsf{cert} = (c_i, d_i)_{i\in[k]}$.

$\mathsf{ABE\text{-}CR^2\text{-}SKL.Vrfy}(\mathsf{vk}, \mathsf{cert})$**:**

- Parse $\mathsf{vk} = \left(\mathsf{skecd.vk} = (x,\theta), \{\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1}\}_{i\in[k]:\theta[i]=1}\right)$ and $\mathsf{cert} = (c_i, d_i)_{i\in[k]}$.
- Output $\top$ if $x[i] = c_i \oplus d_i \cdot (\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1})$ holds for every $i \in [k]$ such that $\theta[i] = 1$ and $\bot$ otherwise.

Let $sk \leftarrow \mathsf{ABE\text{-}CR^2\text{-}SKL}.\mathcal{KG}(\mathsf{msk}, y)$. sk is of the form $(\rho_i)_{i\in[k]}$, where ρ_i is of the following form, where (x,θ) is the verification key of a BB84-based SKECD scheme:

$$\rho_i = \begin{cases} |x[i]\rangle \left|\mathsf{abe.sk}_{i,x[i]}\right\rangle & if\ \ \theta[i] = 0 \\ |0\rangle\,|\mathsf{abe.sk}_{i,0}\rangle + (-1)^{x[i]}\,|1\rangle\,|\mathsf{abe.sk}_{i,1}\rangle & if\ \ \theta[i] = 1, \end{cases}$$

Recall that $\mathsf{abe.sk}_{i,b} = \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, i, t\|b)$ for every $i \in \{2,\ldots,k\}$ and $b \in \{0,1\}$. Moreover, $\mathsf{abe.sk}_{1,b} = \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, 1, t\|b\|y)$ for each $b \in \{0,1\}$.

Decryption correctness. The MI-ABE relation defined in the construction is as follows:

$R(\widetilde{x}\|z, y_1, \cdots, y_k)$**:** Let $\widetilde{x}$ be interpreted as a circuit and z as a k-input circuit.

- Parse $y_1 = t_1\|u_1\|y$.
- Parse $y_i = t_i\|u_i$ for every $i \in 2,\ldots,k$.
- If $t_i \neq t_j$ for some $i, j \in [k]$, output 1. Otherwise, go to the next step.
- If $z(u_1\|\cdots\|u_k) = \bot$ AND $\widetilde{x}(y) = 0$, output 0 (decryptable). Else, output 1.

Clearly, $z := \widetilde{C} \leftarrow \mathsf{CC.Sim}(1^\lambda, \mathsf{pp}_D, 1)$ always outputs $\bot$ and $t_1 = \ldots = t_k$ holds by construction. Hence, the guarantee follows from the decryption correctness of MI-ABE, as long as $\widetilde{x}(y) = 0$ holds, as desired.

Verification correctness. Recall that the verification only checks the Hadamard basis positions, i.e, positions $i \in [k]$ such that $\theta[i] = 1$. Consider the outcome (c_i, d_i) obtained by measuring the state $|0\rangle\,|\mathsf{abe.sk}_{i,0}\rangle + (-1)^{x[i]}\,|1\rangle\,|\mathsf{abe.sk}_{i,1}\rangle$ in the Hadamard basis, where c_i denotes the first bit of the outcome. It is easy to see that $x[i] = c_i \oplus d_i \cdot (\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1})$ is satisfied. Hence, the verification correctness follows.

8.4 Proof of Selective IND-KLA Security

Let $\mathcal{A}$ be an adversary for the selective IND-KLA security of $\mathsf{ABE\text{-}CR^2\text{-}SKL}$. We consider the following sequence of experiments.

$\mathsf{Hyb}_0^{\mathsf{coin}}$**:** This is $\mathsf{Exp}^{\mathsf{sel\text{-}ind\text{-}kla}}_{\mathsf{ABE\text{-}CR^2\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$.

1. $\mathcal{A}$ declares the challenge ciphertext attribute $x^* \in \mathcal{X}$.
2. The challenger $\mathcal{Ch}$ generates $(\mathsf{abe.pk}, \mathsf{abe.msk}) \leftarrow \mathsf{MI\text{-}ABE.Setup}(1^\lambda)$ and $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$, and sends $\mathsf{ek} := \mathsf{abe.pk}$ to $\mathcal{A}$.
3. $\mathcal{A}$ can get access to the following (stateful) oracles, where the list $L_{\mathcal{KG}}$ used by the oracles is initialized to an empty list:

   $O_{\mathcal{KG}}(y)$**:** Given y, it finds an entry of the form (y, vk_y, V_y) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it generates $(sk_y, \mathsf{vk}_y) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$, sends sk_y to $\mathcal{A}$, and adds $(y, \mathsf{vk}_y, \bot)$ to $L_{\mathcal{KG}}$.

$O_{\mathsf{Vrfy}}(y,\mathsf{cert})$: Given (y,cert), it finds an entry (y,vk_y,V_y) from $L_{\mathcal{KG}}$. (If there is no such entry, it returns $\bot$.) It then runs $d \coloneqq \mathsf{Vrfy}(\mathsf{vk}_y,\mathsf{cert})$ and returns d to $\mathcal{A}$. If $V_y=\bot$, it updates the entry into (y,vk_y,d).

4. $\mathcal{A}$ sends $(\mathsf{m}_0^*,\mathsf{m}_1^*)\in\mathcal{M}^2$ to the challenger. If $V_j=\bot$ for some $j\in[q]$, $\mathcal{Ch}$ outputs 0 as the final output of this experiment. Otherwise, $\mathcal{Ch}$ generates $\widetilde{C}^*\leftarrow\mathsf{CC.Sim}(1^\lambda,\mathsf{pp}_D,1)$ and $\mathsf{abe.ct}^*\leftarrow\mathsf{MI\text{-}ABE.Enc}(\mathsf{abe.pk},x^*\|\widetilde{C}^*,\mathsf{m}^*_{\mathsf{coin}})$, and sends $\mathsf{ct}^*\coloneqq\mathsf{abe.ct}^*$ to $\mathcal{A}$.
5. $\mathcal{A}$ outputs coin'. $\mathcal{Ch}$ outputs coin' as the final output of the experiment.

$\mathsf{Hyb}_1^{\mathsf{coin}}$: This is the same as $\mathsf{Hyb}_0^{\mathsf{coin}}$ except that $\widetilde{C}^*$ is generated as $\widetilde{C}^*\leftarrow\mathsf{CC.Obf}(1^\lambda,D[\mathsf{lock}\oplus r,\mathsf{skecd.sk}],\mathsf{lock},0)$, where $\mathsf{lock}\leftarrow\{0,1\}^\lambda$, $r\leftarrow\{0,1\}^\lambda$ and $D[\mathsf{lock}\oplus r,\mathsf{skecd.sk}](x)$ is a circuit that has $\mathsf{lock}\oplus r$ and $\mathsf{skecd.sk}$ hardwired and outputs $\mathsf{lock}\oplus r\oplus\mathsf{SKECD.CDec}(\mathsf{skecd.sk},x)$.

Since lock is chosen at random independently of all other variables, from the security of compute-and-compare obfuscation, we have that $\mathsf{Hyb}_0^{\mathsf{coin}}\approx\mathsf{Hyb}_1^{\mathsf{coin}}$.

$\mathsf{Hyb}_2^{\mathsf{coin}}$: This is the same as $\mathsf{Hyb}_1^{\mathsf{coin}}$ except that $\mathsf{skecd}.ct$ generated as part of $\mathsf{ABE\text{-}CR^2\text{-}SKL}.\mathcal{KG}(\mathsf{msk},y)$ such that $x^*(y)=0$, is generated as $(\mathsf{skecd}.ct,\mathsf{skecd.vk})\leftarrow\mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk},r)$.

In the previous step, we changed the distribution of $\widetilde{C}^*$ used to generate the challenge ciphertext so that $\widetilde{C}^*$ has $\mathsf{skecd.sk}$ hardwired. However, the ciphertext is given to $\mathcal{A}$ after $\mathcal{A}$ deletes all the leased secret keys satisfying the ABE relation, and thus the corresponding ciphertetexts of SKECD. Thus, we can use the security of SKECD to argue that $\mathsf{Hyb}_1^{\mathsf{coin}}\approx\mathsf{Hyb}_2^{\mathsf{coin}}$. Let q be the number of key-queries made to $O_{\mathcal{KG}}$ that satisfy the relation wrt x^*. Consider now $q+1$ hybrids $\mathsf{Hyb}_{1,0}^{\mathsf{coin}},\cdots,\mathsf{Hyb}_{1,q}^{\mathsf{coin}}$ where for every $l\in[q]$, $\mathsf{Hyb}_{1,l}^{\mathsf{coin}}$ is such that the first l of the q keys are generated using SKECD encryptions of r (instead of 0^λ). Notice that $\mathsf{Hyb}_{1,0}^{\mathsf{coin}}\equiv\mathsf{Hyb}_1^{\mathsf{coin}}$ and $\mathsf{Hyb}_{1,q}^{\mathsf{coin}}\equiv\mathsf{Hyb}_2^{\mathsf{coin}}$. Suppose that $\mathsf{Hyb}_{1,l}^{\mathsf{coin}}\not\approx\mathsf{Hyb}_{1,l+1}^{\mathsf{coin}}$ for some $l\in[q]$. Let $\mathcal{D}$ be a distinguisher with non-negligible advantage in distinguishing the hybrids. We will construct the following reduction to the IND-CVA-CD security of SKECD:

Execution of $\mathcal{R}^{\mathcal{D}}$ in $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{R}}(1^\lambda,b)$:

1. The challenger $\mathcal{Ch}$ computes $\mathsf{sk}\leftarrow\mathsf{KG}(1^\lambda)$.
2. $\mathcal{D}$ declares a challenge ciphertext attribute x^* to $\mathcal{R}$. $\mathcal{R}$ generates $(\mathsf{abe.pk},\mathsf{abe.msk})\leftarrow\mathsf{MI\text{-}ABE.Setup}(1^\lambda)$ and sends $\mathsf{ek}\coloneqq\mathsf{abe.pk}$ to $\mathcal{D}$.
3. $\mathcal{R}$ sends $(r_0,r_1)\coloneqq(0^\lambda,r)$ to $\mathcal{Ch}$.
4. $\mathcal{Ch}$ computes $(ct^\star,\mathsf{vk}^\star)\leftarrow\mathcal{E}nc(\mathsf{sk},r_b)$ and sends $ct^\star$ to $\mathcal{R}$.
5. $\mathcal{R}$ simulates the oracle $O_{\mathcal{KG}}(y)$ for $\mathcal{D}$ as follows. Given y, it finds an entry (y,vk_y,V_y) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it proceeds as follows:
   - For the j-th query (of the q satisfying key-queries), if $j\neq l+1$, $\mathcal{R}$ computes $s\mathcal{K}_y$ and vk_y as in $\mathsf{Hyb}_0^{\mathsf{coin}}$, except that the values $(\mathsf{skecd}.ct,\mathsf{skecd.vk})$ are computed using $O_{\mathsf{Enc}}(r)$ for $j\in[l]$ and using $O_{\mathsf{Enc}}(0^\lambda)$ for $j\in\{l+2,\ldots,q\}$.
   - For the j-th query (of the q satisfying key-queries), if $j=l+1$, $\mathcal{R}$ computes $s\mathcal{K}_y$ as in $\mathsf{Hyb}_0^{\mathsf{coin}}$, except that the value $ct^\star$ is used instead of $\mathsf{skecd}.ct$. Consider $\{\mathsf{abe.sk}_{i,0},\mathsf{abe.sk}_{i,1}\}_{i\in[k]:\theta[i]=1}$ computed during the computation of $s\mathcal{K}_y$. $\mathcal{R}$ computes the value $\widetilde{\mathsf{vk}}_y=\{\mathsf{abe.sk}_{i,0}\oplus\mathsf{abe.sk}_{i,1}\}_{i\in[k]}$.
   - If $x^*(y)\neq 0$ (unsatisfying key-queries), $\mathcal{R}$ computes $s\mathcal{K}_y$ and vk_y as in $\mathsf{Hyb}_0^{\mathsf{coin}}$, except that $(\mathsf{skecd}.ct,\mathsf{skecd.vk})$ are computed using $O_{\mathsf{Enc}}(0^\lambda)$.
6. $\mathcal{R}$ simulates the view of oracle $O_{\mathsf{Vrfy}}(y,\mathsf{cert})$ for $\mathcal{D}$ as follows. Given (y,cert), it finds an entry (y,vk_y,V_y) from $L_{\mathcal{KG}}$. If there is no such entry, it returns $\bot$. Otherwise, it proceeds as follows:
   - If $x^*(y)\neq 0$, $\mathcal{R}$ simulates access to O_{Vrfy} as in $\mathsf{Hyb}_0^{\mathsf{coin}}$.
   - If y corresponds to the j-th satisfying key-query to $O_{\mathcal{KG}}$ and $j\neq l+1$, $\mathcal{R}$ simulates access to O_{Vrfy} as in $\mathsf{Hyb}_0^{\mathsf{coin}}$.

- If y corresponds to the j-th satisfying key-query to $O_{\mathcal{KG}}$ and $j = l+1$, $\mathcal{R}$ simulates access to $O_{\mathsf{Vrfy}}(y, \mathsf{cert})$ as follows:
  (a) Parse $\widetilde{\mathsf{vk}}_y = \{\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1}\}_{i\in[k]}$ and $\mathsf{cert} = (c_i, d_i)_{i\in[k]}$.
  (b) Compute $x[i] = c_i \oplus d_i \cdot (\mathsf{abe.sk}_{i,0} \oplus \mathsf{abe.sk}_{i,1})$ for every $i \in [k]$.
  (c) If $O_{\mathsf{Vrfy}}(x) = \mathsf{sk}$, set $d := \top$. Else, set $d := \bot$.
  It returns d to $\mathcal{D}$. If $V_y = \bot$, it updates the entry in $L_{\mathcal{KG}}$ into (y, vk_y, d).

7. $\mathcal{D}$ sends $(m_0^*, m_1^*) \in \mathcal{M}^2$ to $\mathcal{R}$. $\mathcal{R}$ computes ct^* as in Step 4. of $\mathsf{Hyb}_1^{\mathsf{coin}}$ using $\mathsf{skecd.sk} := \mathsf{sk}$ obtained from $\mathcal{CH}$. It sends ct^* to $\mathcal{D}$ if $V_y = \top$ for every entry of the form (y, vk_y, V_y) in $L_{\mathcal{KG}}$ such that $x^*(y) = 0$. Else, it outputs $\bot$.
8. $\mathcal{D}$ guesses a bit b'. $\mathcal{R}$ sends b' to $\mathcal{CH}$.

Notice that the view of $\mathcal{D}$ in the reduction is that of $\mathsf{Hyb}_{1,l}^{\mathsf{coin}}$ if $b = 0$ and $\mathsf{Hyb}_{1,l+1}^{\mathsf{coin}}$ if $b = 1$. Moreover, $\mathcal{D}$ can only distinguish when $V_y = \top$ for all entries $(y, \mathsf{vk}_y, V_y) \in L_{\mathcal{KG}}$ such that $x^*(y) = 0$. This means that V_y corresponding to the $(l+1)$-th satisfying query to $O_{\mathcal{KG}}$ also satisfies $V_y = \top$. Consequently, the corresponding verification check of $\mathcal{CH}$ is also $\top$. Hence, $\mathcal{R}$ succeeds in breaking IND-CVA-CD security of SKECD. Therefore, we have $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$.

We will now bound the distinguishing gap between Hyb_2^0 and Hyb_2^1 using the security of $\mathsf{MI\text{-}ABE}$. Consider a decryption key $s\mathcal{K}_y = (\rho_i)_{i\in[k]}$ such that $x^*(y) = 0$. Recall that a ciphertext of BB84-based SKECD is of the form $|\psi_1\rangle \otimes \cdots \otimes |\psi_k\rangle$, where

$$|\psi_i\rangle = \begin{cases} |x[i]\rangle & \text{if } \theta[i] = 0 \\ |0\rangle + (-1)^{x[i]} |1\rangle & \text{if } \theta[i] = 1 \end{cases}$$

As a result, we have

$$\rho_i = \begin{cases} |x[i]\rangle \left|\mathsf{abe.sk}_{i,x[i]}\right\rangle & \text{if } \theta[i] = 0 \\ |0\rangle |\mathsf{abe.sk}_{i,0}\rangle + (-1)^{x[i]} |1\rangle |\mathsf{abe.sk}_{i,1}\rangle & \text{if } \theta[i] = 1, \end{cases}$$

where $\mathsf{abe.sk}_{1,b} \leftarrow \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, 1, t_y \| b \| y)$ for each $b \in \{0,1\}$ and $\mathsf{abe.sk}_{i,b}$ is generated as $\mathsf{abe.sk}_{i,b} \leftarrow \mathsf{MI\text{-}ABE.KG}(\mathsf{abe.msk}, i, t_y \| b)$ for each $i \in \{2, \ldots, k\}$ and $b \in \{0,1\}$. Here, t_y is a random value that is common for each of the k "slots" of the decryption-key $s\mathcal{K}_y$. Notice first that no two values t_y, t_w are equal for t_w corresponding to some $s\mathcal{K}_w$ except with negligible probability. Consequently, due to the defined relation R and the selective security of $\mathsf{MI\text{-}ABE}$, any set of k secret keys is "decryptable" only if they correspond to the same decryption key $s\mathcal{K}_y$. Now, we notice that $\mathsf{abe.sk}_{i,1-x[i]}$ is not given to $\mathcal{A}$ for any $i \in [k]$ such that $\theta[i] = 0$. This means that for any set of k $\mathsf{MI\text{-}ABE}$ keys corresponding to the decryption-key $s\mathcal{K}_y$, the attributes $(x'[1], \cdots, x'[k])$ satisfy $x'[i] = x[i]$ for all $i : \theta[i] = 0$. Consequently, the classical decryption property (See Definition 3.18) of the BB84-based SKE-CD scheme guarantees that $\mathsf{lock} \oplus r \oplus \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, x') = \mathsf{lock}$ in these hybrids. This means that $\widetilde{C}^*(x') = 0$ (instead of $\bot$). As a result, any subset of $\mathsf{MI\text{-}ABE}$ keys of size k given to $\mathcal{A}$ is a "non-decryptable" set in the hybrids Hyb_2^0 and Hyb_2^1. It is easy to see now that we can reduce to the selective security of $\mathsf{MI\text{-}ABE}$. Specifically, the reduction specifies the target attribute $x^* \| \widetilde{C}^*$ where $\widetilde{C}^* \leftarrow \mathsf{CC.Obf}(1^\lambda, D[\mathsf{lock} \oplus r, \mathsf{skecd.sk}], \mathsf{lock}, 0)$ and simulates the view for $\mathcal{A}$ by querying the key-generation oracle of $\mathsf{MI\text{-}ABE}$ accordingly. Note that the reduction can handle verification queries since it can obtain both $\mathsf{abe.sk}_{i,0}$ and $\mathsf{abe.sk}_{i,1}$ for every $i \in [k]$ such that $\theta[i] = 1$ that are sufficient to perform the verification. (Especially, it does not need $\mathsf{abe.sk}_{i,1-x[i]}$ for $i \in [k]$ such that $\theta[i] = 0$ for verification.) We now state the following theorem:

Theorem 8.7. *There exists an ABE-CR^2-SKL scheme satisfying selective IND-KLA Security, assuming the existence of a selectively-secure Multi-Input ABE scheme for polynomial-arity, Compute-and-Compare Obfuscation, and a BB84-based SKE-CD scheme.*

9 Adaptively-Secure IBE-CR-SKL

In this section, we show that we can obtain an *adaptively* IND-KLA secure *IBE*-CR-SKL scheme assuming the *polynomial* hardness of the LWE assumption.

9.1 Definitions

Definition 9.1 (IBE with Secure Key Leasing). *An IBE-SKL scheme* IBE-CR-SKL *is a tuple of six algorithms* $(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$. *Below, let* $\mathcal{ID} = \{\mathcal{ID}_\lambda\}_\lambda$ *be the identity space.*

$\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{msk})$**:** *The setup algorithm takes a security parameter* 1^λ*, and outputs a public key* pk *and master secret key* msk.

$\mathcal{KG}(\mathsf{msk}, \mathsf{id}) \to (\mathit{dk}_{\mathsf{id}}, \mathsf{vk}_{\mathsf{id}})$**:** *The key generation algorithm takes a master secret key* msk *and an identity* $\mathsf{id} \in \mathcal{ID}$*, and outputs a decryption key* $\mathit{dk}_{\mathsf{id}}$ *and a verification key* $\mathsf{vk}_{\mathsf{id}}$.

$\mathsf{Enc}(\mathsf{pk}, \mathsf{id}, \mathsf{m}) \to \mathsf{ct}$**:** *The encryption algorithm takes a public key* pk*, an identity* $\mathsf{id} \in \mathcal{ID}$*, and a plaintext* m*, and outputs a ciphertext* ct.

$\mathcal{Dec}(\mathit{dk}, \mathsf{ct}) \to z$**:** *The decryption algorithm takes a decryption key* dk*, and a ciphertext* ct *and outputs a value* $z \in \{\bot\} \cup \{0,1\}^\ell$.

$\mathcal{Vrfy}(\mathsf{vk}, \mathit{dk}') \to \top/\bot$**:** *The verification algorithm takes a verification key* vk *and a quantum state* dk'*, and outputs* $\top$ *or* $\bot$.

Decryption correctness: *For every* $\mathsf{id} \in \mathcal{ID}$*, we have*

$$\Pr\left[\mathcal{Dec}(\mathit{dk}_{\mathsf{id}}, \mathsf{ct}_{\mathsf{id}}) = \mathsf{m} \;\middle|\; \begin{array}{l}(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{dk}_{\mathsf{id}}, \mathsf{vk}_{\mathsf{id}}) \leftarrow \mathcal{KG}(\mathsf{msk}, \mathsf{id}) \\ \mathsf{ct}_{\mathsf{id}} \leftarrow \mathsf{Enc}(\mathsf{pk}, \mathsf{id}, \mathsf{m})\end{array}\right] = 1 - \mathsf{negl}(\lambda).$$

Verification correctness: *For every* $\mathsf{id} \in \mathcal{ID}$*, we have*

$$\Pr\left[\mathcal{Vrfy}(\mathsf{vk}_{\mathsf{id}}, \mathit{dk}_{\mathsf{id}}) = \top \;\middle|\; \begin{array}{l}(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda) \\ (\mathit{dk}_{\mathsf{id}}, \mathsf{vk}_{\mathsf{id}}) \leftarrow \mathcal{KG}(\mathsf{msk}, \mathsf{id})\end{array}\right] = 1 - \mathsf{negl}(\lambda).$$

Definition 9.2 (Adaptive Indistinguishability against Key Leasing Attacks). *We say that an IBE-SKL scheme* IBE-CR-SKL *is secure against adaptive indistinguishability against key leasing attacks (Ada-IND-KLA), if it satisfies the following requirement, formalized from the experiment* $\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathcal{A},\mathsf{IBE\text{-}CR\text{-}SKL}}(1^\lambda, \mathsf{coin})$ *between an adversary* $\mathcal{A}$ *and a challenger:*

1. *At the beginning, the challenger runs* $(\mathsf{pk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$ *and initialize the list* $L_{\mathcal{KG}}$ *to be an empty set. Throughout the experiment,* $\mathcal{A}$ *can access the following oracles.*

   $O_{\mathcal{KG}}(\mathsf{id})$**:** *Given* $\mathsf{id} \in \mathcal{ID}$*, it finds entries of the form* $(\mathsf{id}, 1, \mathsf{vk}_{\mathsf{id},1}, V_1), \ldots, (\mathsf{id}, i-1, \mathsf{vk}_{\mathsf{id},i-1}, V_{i-1})$ *from* $L_{\mathcal{KG}}$ *for some* $i \geq 2$*. If there is no such entry, set* $i := 1$*. It then generates* $(\mathit{dk}_{\mathsf{id},i}, \mathsf{vk}_{\mathsf{id},i}) \leftarrow \mathcal{KG}(\mathsf{msk}, \mathsf{id})$*, sends* $\mathit{dk}_{\mathsf{id},i}$ *to* $\mathcal{A}$*, and adds* $(\mathsf{id}, i, \mathsf{vk}_{\mathsf{id},i}, \bot)$ *to* $L_{\mathcal{KG}}$.

   $O_{\mathcal{Vrfy}}(\mathsf{id}, i, \mathit{dk}')$**:** *Given* $(\mathsf{id}, i, \mathit{dk}')$*, it finds an entry* $(\mathsf{id}, i, \mathsf{vk}, V)$ *from* $L_{\mathcal{KG}}$*. (If there is no such entry, it returns* $\bot$*.) It then runs* $d := \mathcal{Vrfy}(\mathsf{vk}, \mathit{dk}')$ *and returns* d *to* $\mathcal{A}$*. If* $V = \bot$*, it updates the entry into* $(\mathsf{id}, i, \mathsf{vk}, d)$.

2. *When* $\mathcal{A}$ *sends* $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$ *to the challenger, the challenger checks if for any entry* $(\mathsf{id}^*, i, \mathsf{vk}, V)$ *in* $L_{\mathcal{KG}}$*, it holds that* $V = \top$*. If so, the challenger generates* $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{pk}, \mathsf{id}^*, \mathsf{m}_{\mathsf{coin}})$ *and sends* ct^* *to* $\mathcal{A}$*. Otherwise, the challenger outputs* 0.

3. $\mathcal{A}$ *continues to make queries to* $O_{\mathcal{KG}}(\cdot)$ *and* $O_{\mathcal{Vrfy}}(\cdot,\cdot,\cdot)$*. However,* $\mathcal{A}$ *is not allowed to send the target identity* id^* *to* $O_{\mathcal{KG}}$.

4. *$\mathcal{A}$ outputs a guess* coin' *for* coin. *The challenger outputs* coin' *as the final output of the experiment.*

For any QPT $\mathcal{A}$, it holds that

$$\mathsf{Adv}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{IBE\text{-}CR\text{-}SKL},\mathcal{A}}(\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{IBE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda,0)\to 1\right]-\Pr\left[\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{IBE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda,1)\to 1\right]\right|\le \mathsf{negl}(\lambda).$$

9.2 Construction

Construction overview. Our construction idea is based on the compiler by Agrawal et al. [AKN+23]. Their compiler shows how to obtain adaptively secure ABE-SKL from a PKE-SKL scheme, a garbled circuit scheme, and a classical adaptively secure ABE scheme. We observe that an analogous construction and security claim also hold for IBE-SKL. Concretely, we use our PKE-CR-SKL scheme and classical adaptively secure IBE scheme, in place of the non-collusion-resistant PKE-SKL scheme and the classical adaptively secure ABE scheme in their construction.

An important point is that the compiler of Agrawal et al. was proved only in the "1-bounded setting", where the adversary is allowed to make only a single decryption-key query satisfying the target attribute. We observe that this restriction can be removed in the IBE-SKL setting. That is, we can allow the adversary to obtain arbitrary many keys corresponding to the target identity as well, provided that the underlying PKE-SKL scheme satisfies collusion-resistance.

To generate a decryption key for an identity id, we first generate a PKE-CR-SKL key pair $(\mathsf{pke.ek}_{\mathsf{id}},\mathsf{pke.msk}_{\mathsf{id}})$ by running the setup algorithm of PKE-CR-SKL with randomness $\mathsf{PRF}_{\mathsf{K}}(\mathsf{id})$, where PRF is a PRF. Thus, the same identity id always determines the same key pair $(\mathsf{pke.ek}_{\mathsf{id}},\mathsf{pke.msk}_{\mathsf{id}})$. We then generate a fresh quantum decryption key $\mathsf{pke}.d\mathcal{K}$ by running $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{pke.msk}_{\mathsf{id}})$. In addition, the user receives IBE secret keys for identity id only at the positions corresponding to $\mathsf{pke.ek}_{\mathsf{id}}$.

For encryption, we garble the encryption circuit of $\mathsf{PKE\text{-}CR\text{-}SKL}$ with the message hardwired into the circuit, and encrypt the garbled-circuit labels under identity id using the IBE scheme. A user holding the decryption key for id can recover exactly the labels corresponding to $\mathsf{pke.ek}_{\mathsf{id}}$, evaluate the garbled circuit on these labels, and thereby obtain a PKE-CR-SKL ciphertext encrypted under $\mathsf{pke.ek}_{\mathsf{id}}$.

Although the same id yields the same encryption key $\mathsf{pke.ek}_{\mathsf{id}}$ because of the PRF-based generation, each invocation of key generation produces a fresh quantum decryption key, since $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{pke.msk}_{\mathsf{id}})$ is run independently each time. Therefore, by relying on the collusion resistance of $\mathsf{PKE\text{-}CR\text{-}SKL}$, we can handle multiple decryption keys for the same target identiy and thereby overcome the "1-bounded" limitation in the compiler of Agrawal et al.

We stress that this way of removing the 1-bounded restriction is specific to the IBE setting. The analogous garbled-circuit approach does not directly apply to ABE, because ciphertext attributes and key attributes are asymmetric. In particular, an ABE adversary may receive many different decryption keys whose attributes satisfy the challenge ciphertext attribute.

Notation. In this section, for PPT algorithm Alg, $\mathsf{Alg}(x;r)$ denotes we run Alg with input x and explicit randomness r.

Construction. We construct an IBE-SKL scheme $\mathsf{IBE\text{-}CR\text{-}SKL}=(\mathsf{Setup},\mathcal{KG},\mathsf{Enc},\mathcal{Dec},\mathcal{Vrfy})$ with Ada-IND-KLA security whose message space is $\{0,1\}^\ell$ by using the following building blocks.

- IND-KLA secure PKE-SKL $\mathsf{PKE\text{-}CR\text{-}SKL}.(\mathsf{Setup},\mathcal{KG},\mathsf{Enc},\mathcal{Dec},\mathcal{Vrfy})$. Without loss of generality, we assume that $\mathsf{pke.ek}\in\{0,1\}^{\ell_{\mathsf{ek}}}$ and the randomness space used by $\mathsf{PKE\text{-}CR\text{-}SKL.Enc}$ is $\{0,1\}^{\ell_{\mathsf{rand}}}$ for some $\ell_{\mathsf{ek}}(\lambda)$ and $\ell_{\mathsf{rand}}(\lambda)$. We also assume that the message space of $\mathsf{PKE\text{-}CR\text{-}SKL}$ is $\{0,1\}^\ell$.
- Adaptively secure IBE $\mathsf{IBE}.(\mathsf{Setup},\mathsf{KG},\mathsf{Enc},\mathsf{Dec})$ with message space $\{0,1\}^\lambda$.
- A garbling scheme $\mathsf{GC}=(\mathsf{Grbl},\mathsf{GCEval})$. Without loss of generality, we assume that the labels of GC are in $\{0,1\}^\lambda$.

$\mathsf{Setup}(1^\lambda)$**:**

- Chooses $\mathsf{K}\leftarrow\{0,1\}^\lambda$.
- For $i\in[\ell_{\mathsf{ek}}]$ and $b\in\{0,1\}$, run $(\mathsf{ibe.pk}_{i,b},\mathsf{ibe.msk}_{i,b})\leftarrow\mathsf{IBE.Setup}(1^\lambda)$.

- Output $\mathsf{pk} := \{\mathsf{ibe.pk}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}$ and $\mathsf{msk} := (\{\mathsf{ibe.msk}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \mathsf{K})$.

$\mathcal{KG}(\mathsf{msk}, \mathsf{id})$**:**

- Generate $(\mathsf{pke.ek}_{\mathsf{id}}, \mathsf{pke.msk}_{\mathsf{id}}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \mathsf{PRF}_{\mathsf{K}}(\mathsf{id}))$.
- Run $(\mathsf{pke}.d\mathcal{K}, \mathsf{pke.vk}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{pke.msk}_{\mathsf{id}})$.
- Run $\mathsf{ibe.sk}_i \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}_{i,\mathsf{pke.ek}_{\mathsf{id}}[i]}, \mathsf{id})$ for $i \in [\ell_{\mathsf{ek}}]$, where $\mathsf{pke.ek}_{\mathsf{id}}[i]$ denotes the i-th bit of the binary string $\mathsf{pke.ek}_{\mathsf{id}}$.
- Output $d\mathcal{K} := (\{\mathsf{ibe.sk}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \mathsf{pke.ek}_{\mathsf{id}}, \mathsf{pke}.d\mathcal{K})$ and $\mathsf{vk} := \mathsf{pke.vk}$.

$\mathsf{Enc}(\mathsf{pk}, \mathsf{id}, \mathsf{m})$**:**

- Choose $\mathsf{R} \leftarrow \{0,1\}^{\ell_{\mathsf{rand}}}$.
- Construct circuit $E[\mathsf{m}, \mathsf{R}]$, which is a circuit that takes as input an encryption key $\mathsf{pke.ek}$ of PKE-CR-SKL and outputs $\mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}, \mathsf{m}; \mathsf{R})$.
- Compute $(\{\mathsf{lab}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \widetilde{E}) \leftarrow \mathsf{Grbl}(1^\lambda, E[\mathsf{m}, \mathsf{R}])$.
- Run $\mathsf{ibe.ct}_{i,b} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,b}, \mathsf{id}, \mathsf{lab}_{i,b})$ for $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$.
- Output $\mathsf{ct} := (\{\mathsf{ibe.ct}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \widetilde{E})$.

$\mathcal{D}ec(d\mathcal{K}, \mathsf{ct})$**:**

- Parse $d\mathcal{K} = (\{\mathsf{ibe.sk}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \mathsf{pke.ek}, \mathsf{pke}.d\mathcal{K})$ and $\mathsf{ct} = (\{\mathsf{ibe.ct}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \widetilde{E})$.
- Compute $\mathsf{lab}_i \leftarrow \mathsf{IBE.Dec}(\mathsf{ibe.sk}_i, \mathsf{ibe.ct}_{i,\mathsf{pke.ek}[i]})$ for $i \in [\ell_{\mathsf{ek}}]$.
- Compute $\mathsf{pke.ct} = \mathsf{GCEval}(\widetilde{E}, \{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]})$.
- Compute and output $\mathsf{m}' \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{D}ec(\mathsf{pke}.d\mathcal{K}, \mathsf{pke.ct})$.

$\mathcal{V}rfy(\mathsf{vk}, d\mathcal{K}')$**:**

- Parse $\mathsf{vk} = \mathsf{pke.vk}$ and $d\mathcal{K}' = (\{\mathsf{ibe.sk}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \mathsf{pke.ek}, \mathsf{pke}.d\mathcal{K}')$.
- Compute and output $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}, \mathsf{pke}.d\mathcal{K}')$.

We show that the scheme satisfies decryption correctness. First, by the correctness of IBE, the decryption algorithm correctly recovers the labels of $\widetilde{E}$ corresponding to the input pke.ek. Then, by the correctness of GC, the ciphertext pke.ct obtained from the garbled-circuit evaluation is equal to $\mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}, \mathsf{m}; \mathsf{R})$. Finally, the message m is recovered in the last step by the correctness of PKE-CR-SKL. Verification correctness follows directly from the verification correctness of PKE-CR-SKL.

9.3 Proof of Adaptive IND-KLA Security

Theorem 9.3. *If* IBE *is adaptively secure,* GC *is secure, and* PKE-CR-SKL *is IND-KLA secure, then* IBE-CR-SKL *above is Ada-IND-KLA secure.*

Proof of Theorem 9.3. Let Q be the upper bound on the number of key queries to $O_{\mathcal{KG}}$ before the challenge phase. We define a sequence of hybrid games.

Hyb_0**:** This game is identical to $\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{IBE},\mathcal{A}}(1^\lambda, 0)$. More specifically, it is defined as follows.

1. The challenger samples $\mathsf{K} \leftarrow \{0,1\}^\lambda$ and generates $(\mathsf{ibe.pk}_{i,b}, \mathsf{ibe.msk}_{i,b}) \leftarrow \mathsf{IBE.Setup}(1^\lambda)$ for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$. It sends $\mathsf{pk} := \{\mathsf{ibe.pk}_{i,b}\}_{i,b}$ to the adversary $\mathcal{A}$. The challenger also initializes the list $L_{\mathcal{KG}}$ as the empty set. The adversary $\mathcal{A}$ is given access to the following oracles.

$O_{\mathcal{KG}}(\mathsf{id}^{(j)})$**:** On the j-th query $\mathsf{id}^{(j)}$, it finds entries of the form $(\mathsf{id}^{(j)}, 1, \mathsf{vk}_1, V_1), \ldots (\mathsf{id}^{(j)}, n-1, \mathsf{vk}_{n-1}, V_{n-1})$. It then generates $(\mathsf{pke.ek}^{(j)}, \mathsf{pke.msk}^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \mathsf{PRF}_{\mathsf{K}}(\mathsf{id}^{(j)}))$ and $(\mathsf{pke.}d\mathcal{K}_n^{(j)}, \mathsf{pke.vk}_n^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{pke.msk}^{(j)})$ For every $i \in [\ell_{\mathsf{ek}}]$, it also computes $\mathsf{ibe.sk}_i^{(j)} \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}_{i,\mathsf{pke.ek}^{(j)}[i]}, \mathsf{id}^{(j)})$, where $\mathsf{pke.ek}[i]$ is the i-th bit of the binary string $\mathsf{pke.ek}$. The challenger sends $d\mathcal{K}^{(j)} := (\{\mathsf{ibe.sk}_i^{(j)}\}_i, \mathsf{pke.ek}^{(j)}, \mathsf{pke.}d\mathcal{K}_n^{(j)})$ to $\mathcal{A}$, and adds $(\mathsf{id}^{(j)}, n, \mathsf{pke.vk}_n^{(j)}, \bot)$ to $L_{\mathcal{KG}}$.

$O_{\mathcal{V}rfy}(\mathsf{id}, n, d\mathcal{K}')$**:** On input $(\mathsf{id}, n, d\mathcal{K}')$, it finds an entry $(\mathsf{id}, n, \mathsf{vk}_n, V_n)$ from $L_{\mathcal{KG}}$ and parses $d\mathcal{K}' = (\{\mathsf{ibe.sk}'_i\}_i, \mathsf{pke.ek}, \mathsf{pke.}d\mathcal{K}')$. If there is no such entry, it returns $\bot$. It parses $\mathsf{vk}_n = \mathsf{pke.vk}$ and computes $d := \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}, \mathsf{pke.}d\mathcal{K}')$. It returns d to $\mathcal{A}$. Finally, if $V_n = \bot$, it updates the entry $(\mathsf{id}, n, \mathsf{vk}_n, d)$.

2. When $\mathcal{A}$ sends $(\mathsf{id}^*, m_0, m_1)$ to the challenger, the challenger checks whether there exists an entry $(\mathsf{id}^*, n, \mathsf{vk}_n, V_n)$ in $L_{\mathcal{KG}}$ such that $V_n = \bot$. If such an entry exists for some $n \in [Q]$, the challenger aborts the game and outputs 0. Otherwise, the challenger generates $(\{\mathsf{lab}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \widetilde{E}) \leftarrow \mathsf{Grbl}(1^\lambda, E[\mathsf{m}_0, \mathsf{R}])$ and computes $\mathsf{ibe.ct}_{i,b} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,b}, \mathsf{id}^*, \mathsf{lab}_{i,b})$ for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$. It then sends $\mathsf{ct}^* := (\{\mathsf{ibe.ct}_{i,b}\}_{i,b}, \widetilde{E})$ to $\mathcal{A}$.
3. The adversary $\mathcal{A}$ continues to make queries to $O_{\mathcal{KG}}(\cdot)$ and $O_{\mathcal{V}rfy}(\cdot,\cdot)$. However, $\mathcal{A}$ is not allowed to query $O_{\mathcal{KG}}$ on the target identity id^*.
4. The adversary $\mathcal{A}$ outputs a guess coin' for coin. The challenger outputs coin' as the final output of the experiment.

Hyb_1**:** This game proceeds as Hyb_0, with the following modification. At the outset, the challenger samples a random index $\tilde{j} \leftarrow [Q]$. Once $\mathcal{A}$ submits $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$, and just before generating the challenge ciphertext, the challenger finds the smallest index $j^* \in [Q]$ such that there exists an entry $(\mathsf{id}^{(j^*)}, n, \mathsf{pke.vk}_n^{(j^*)}, V_n)$ in $L_{\mathcal{KG}}$ with $\mathsf{id}^* = \mathsf{id}^{(j^*)}$. If no such entry exists, we define $j^* := 1$. The challenger then checks whether $\tilde{j} = j^*$. If this equality holds, the challenger continues the execution exactly as in Hyb_0 until $\mathcal{A}$ outputs its guess. If not, the challenger aborts the game and outputs 0.

Since $\tilde{j}$ is sampled independently of the view of $\mathcal{A}$, and since Hyb_1 can output 1 only in the event that $\tilde{j} = j^*$, we have $\Pr[\mathsf{Hyb}_1 = 1] = \Pr[\mathsf{Hyb}_0 = 1]/Q$.

Hyb_2**:** This game is the same as Hyb_1 except that $O_{\mathcal{KG}}$ chooses random $\rho^{(j)}$ for $\mathsf{id}^{(j)}$ and uses it to generate $(\mathsf{pke.ek}^{(j)}, \mathsf{pke.msk}^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \rho^{(j)})$ instead of using K. It stores $(\mathsf{id}^{(j)}, \rho^{(j)})$ and uses the same randomness for the same identity $\mathsf{id}^{(j)}$. We claim that this game is indistinguishable from the previous one. This immediately follows from the security of the PRF since K is never used in any other place. Hence, we obtain $|\Pr[\mathsf{Hyb}_1 = 1] - \Pr[\mathsf{Hyb}_2 = 1]| = \mathsf{negl}(\lambda)$.

Hyb_3**:** This game differs from Hyb_2 only in how the ciphertexts $\{\mathsf{ibe.ct}_{i,b}\}_{i,b}$ are generated. Specifically, for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$, the challenger computes

$$\mathsf{ibe.ct}_{i,b} = \mathsf{ibe.ct}_{i,b} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,b}, \mathsf{id}^*, \mathsf{lab}_{i,\mathsf{pke.ek}^{(j^*)}[i]}).$$

Notice that this modification changes the encrypted labels only for the ciphertexts indexed by $(i, 1 \oplus \mathsf{pke.ek}^{(j^*)}[i])$. The adversary $\mathcal{A}$ cannot detect this change, because it receives no IBE secret key that enables decryption of these ciphertexts under the corresponding public keys. Indeed, for identity $\mathsf{id}^* = \mathsf{id}^{(j^*)}$, the encryption key is uniquely determined as $\mathsf{pke.ek}^* = \mathsf{pke.ek}^{(j^*)}$, where $(\mathsf{pke.ek}^*, \mathsf{pke.msk}^*) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \rho^{(j^*)})$. For the corresponding key query, the adversary obtains IBE secret keys only for public keys indexed by $(i, \mathsf{pke.ek}^*[i])$. Therefore, the ciphertexts whose encrypted labels are modified are precisely those for which $\mathcal{A}$ has no corresponding IBE secret key for the target identity id^*. Hence, by the adaptive security of IBE, we obtain $|\Pr[\mathsf{Hyb}_2 = 1] - \Pr[\mathsf{Hyb}_3 = 1]| = \mathsf{negl}(\lambda)$. See Lemma 9.4 for the details.

Hyb_4**:** This game differs from Hyb_3 only in the generation of ct^*. More specifically, to generate ct^*, the challenger first computes $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E}) \leftarrow \mathsf{Sim.GC}(1^\lambda, \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_0; \mathsf{R}))$ and then, for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$, computes $\mathsf{ibe.ct}_{i,b} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,b}, \mathsf{id}^*, \mathsf{lab}_i)$.

We claim that this game is computationally indistinguishable from Hyb_3. It is enough to show that $(\{\mathsf{lab}_{i,\mathsf{pke.ek}^*[i]}\}_i, \widetilde{E})$ generated as part of $(\{\mathsf{lab}_{i,b}\}_{i,b}, \widetilde{E}) \leftarrow \mathsf{Grbl}(1^\lambda, E[\mathsf{m}_0, \mathsf{R}])$ is computationally indistinguishable from $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E})$ generated as $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E}) \leftarrow \mathsf{Sim.GC}(1^\lambda, \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_0; \mathsf{R}))$. This follows directly from the security of the garbled circuit, because the definition of E gives

$$E[\mathsf{m}_0, \mathsf{R}](\mathsf{pke.ek}^*) = \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_0; \mathsf{R})$$

Therefore, $|\Pr[\mathsf{Hyb}_3 = 1] - \Pr[\mathsf{Hyb}_4 = 1]| = \mathsf{negl}(\lambda)$.

Hyb_5**:** This game differs from Hyb_4 only in the input given to the garbled circuit simulator. Namely, the challenger computes the simulated garbled circuit and labels as $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E}) \leftarrow \mathsf{Sim.GC}(1^\lambda, \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_1; \mathsf{R}))$ instead of $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E}) \leftarrow \mathsf{Sim.GC}(1^\lambda, \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_0; \mathsf{R}))$.

To prove that $|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]| = \mathsf{negl}(\lambda)$, it is enough to argue that $\mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_0; \mathsf{R})$ and $\mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}^*, \mathsf{m}_1; \mathsf{R})$ are computationally indistinguishable from the viewpoint of $\mathcal{A}$, conditioned on the event that, before receiving the challenge ciphertext, $\mathcal{A}$ makes $O_{\mathcal{V}rfy}$ output $\top$ on every relevant verification query for the target identity $\mathsf{id}^{(j^*)}$.

This indistinguishability is guaranteed by the security of PKE-CR-SKL. Indeed, if $\mathcal{A}$ passes the verification oracle $O_{\mathcal{V}rfy}$ for the target identity before the challenge ciphertext is generated, then it has submitted valid returned keys $\mathsf{pke}.dk'_n$ satisfying $\mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}_n^{(j^*)}, \mathsf{pke}.dk'_n) = \top$ for the relevant indices n. Thus, the challenge ciphertext is produced only after the corresponding leased PKE-SKL decryption keys have been successfully returned, which is precisely the setting covered by the security of PKE-CR-SKL. To formalize this intuition, the reduction must embed the public key of the PKE-CR-SKL security experiment into the answer to j^*-th key-generation query. Since the reduction algorithm does not know j^* until $\mathcal{A}$ submits $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$, it must guess this index in advance. The modification introduced in Hyb_1 incorporates this guess into the game, thereby enabling the reduction. We refer to Lemma 9.5 for the formal proof that $|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]| = \mathsf{negl}(\lambda)$.

Hyb_6**:** This game is identical to $\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{IBE},\mathcal{A},1}(1^\lambda, 1)$.

From the above discussion, we have

$$\begin{aligned} |Q\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_0 = 1]| &= Q|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_1 = 1]| \\ &\le Q \sum_{i\in[0,3]} |\Pr[\mathsf{Hyb}_{i+1} = 1] - \Pr[\mathsf{Hyb}_i = 1]| \le \mathsf{negl}(\lambda), \end{aligned} \tag{1}$$

where the first equality follows from $\Pr[\mathsf{Hyb}_1 = 1] = \Pr[\mathsf{Hyb}_0 = 1]/Q$, and the last inequality uses $Q = \mathrm{poly}(\lambda)$. We next observe that Hyb_6 is the same as Hyb_0, and Hyb_5 is the same as Hyb_4, except that m_1 is used for encryption instead of m_0. Therefore, by considering the analogous sequence of games in reverse order with m_0 replaced by m_1, we obtain $|Q\Pr[\mathsf{Hyb}_5 = 1] - \Pr[\mathsf{Hyb}_6 = 1]| \le \mathsf{negl}(\lambda)$. We therefore have

$$\begin{aligned} &\left|\mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{1ABE},\mathcal{A},1}(1^\lambda, 0) - \mathsf{Exp}^{\mathsf{ada\text{-}ind\text{-}kla}}_{\mathsf{1ABE},\mathcal{A},1}(1^\lambda, 1)\right| \\ &= |\Pr[\mathsf{Hyb}_0 = 1] - \Pr[\mathsf{Hyb}_6 = 1]| \\ &\le |\Pr[\mathsf{Hyb}_0 = 1] - Q\Pr[\mathsf{Hyb}_4 = 1]| + Q|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]| + |Q\Pr[\mathsf{Hyb}_5 = 1] - \Pr[\mathsf{Hyb}_6 = 1]| \\ &\le \mathsf{negl}(\lambda) \end{aligned}$$

as desired. It remains to prove Lemmata 9.4 and 9.5.

Lemma 9.4. $|\Pr[\mathsf{Hyb}_2 = 1] - \Pr[\mathsf{Hyb}_3 = 1]| = \mathsf{negl}(\lambda)$ *if* IBE *is adaptively secure.*

Proof. This can be reduced to the adaptive security of IBE by a standard hybrid argument in which we modify the generation of the ciphertexts $\mathsf{ibe.ct}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]}$ one index $i \in [\ell_{\mathsf{ek}}]$ at a time. More precisely, the reduction works as follows.

We define additional hybrids $\mathsf{Hyb}_{2.k}$ for $k \in [\ell_{\mathsf{ek}}+1]$.

$\mathsf{Hyb}_{2.k}$**:** This game is identical to Hyb_2 except that, for every $i \in [\ell_{\mathsf{ek}}]$, the ciphertext $\mathsf{ibe.ct}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]}$ is generated as

$$\mathsf{ibe.ct}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]} \leftarrow \begin{cases} \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]}, \mathsf{id}^*, \mathsf{lab}_{i,\mathsf{pke.ek}^{(j^*)}[i]}) & i < k \\ \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]}, \mathsf{id}^*, \mathsf{lab}_{i,1\oplus\mathsf{pke.ek}^{(j^*)}[i]}) & i \geq k. \end{cases} \tag{2}$$

Clearly, $\mathsf{Hyb}_2 = \mathsf{Hyb}_{2.1}$ and $\mathsf{Hyb}_3 = \mathsf{Hyb}_{2.(\ell_{\mathsf{ek}}+1)}$. Thus, it suffices to prove that, for every $k \in [\ell_{\mathsf{ek}}]$, $\left|\Pr\left[\mathsf{Hyb}_{2.(k+1)} = 1\right] - \Pr[\mathsf{Hyb}_{2.k} = 1]\right| = \mathsf{negl}(\lambda)$. The only difference between $\mathsf{Hyb}_{2.k}$ and $\mathsf{Hyb}_{2.(k+1)}$ is the way of generating $\mathsf{ibe.ct}_{k,1\oplus\mathsf{pke.ek}^{(j^*)}[k]}$. To prove the above claim, we construct an adversary $\mathcal{B}$ against the adaptive security of IBE as follows.

$\mathcal{B}(\mathsf{ibe.pk})$**:** The adversary $\mathcal{B}$ works as follows.

1. It samples $\tilde{j} \leftarrow [Q]$.
2. It samples $\rho^{(\tilde{j})}$ and generates $(\mathsf{pke.ek}^{(\tilde{j})}, \mathsf{pke.msk}^{(\tilde{j})}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \rho^{(\tilde{j})})$.
3. For every $(i,b) \in [\ell_{\mathsf{ek}}] \times \{0,1\} \setminus \{(k, 1\oplus\mathsf{pke.ek}^{(\tilde{j})}[k])\}$, it generates $(\mathsf{ibe.pk}_{i,b}, \mathsf{ibe.msk}_{i,b}) \leftarrow \mathsf{IBE.Setup}(1^\lambda)$. It sets $\mathsf{ibe.pk}_{k,1\oplus\mathsf{pke.ek}^{(\tilde{j})}[k]} := \mathsf{ibe.pk}$ and sends $\mathsf{pk} := \{\mathsf{ibe.pk}_{i,b}\}_{i,b}$ to $\mathcal{A}$.
4. It initializes the list $L_{\mathcal{KG}}$ as the empty set and simulates the following oracles for $\mathcal{A}$.

   $O_{\mathcal{KG}}(\mathsf{id}^{(j)})$**:** On the j-th query $\mathsf{id}^{(j)}$, where $j \in [Q]$, $\mathcal{B}$ first finds entries of the form $(\mathsf{id}^{(j)}, 1, \mathsf{vk}_1, V_1), \ldots (\mathsf{id}^{(j)}, n-1, \mathsf{vk}_{n-1}, V_{n-1})$ in $L_{\mathcal{KG}}$. If $\mathsf{id}^{(j)}$ has not been assigned randomness yet, $\mathcal{B}$ assigns randomness $\rho_{\mathsf{id}^{(j)}}$ to this identity. In particular, if $j = \tilde{j}$, then it sets $\rho_{\mathsf{id}^{(j)}} := \rho^{(\tilde{j})}$. Otherwise, it samples fresh randomness $\rho_{\mathsf{id}^{(j)}}$. If $\mathsf{id}^{(j)}$ has already been assigned randomness, $\mathcal{B}$ reuses the same value $\rho_{\mathsf{id}^{(j)}}$. It then generates $(\mathsf{pke.ek}^{(j)}, \mathsf{pke.msk}^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \rho^{(j)})$ and $(\mathsf{pke}.d\mathcal{K}_n^{(j)}, \mathsf{pke.vk}_n^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{pke.msk}^{(j)})$ and $\mathsf{ibe.sk}_i^{(j)} \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}_{i,\mathsf{pke.ek}^{(j)}[i]}, \mathsf{id}^{(j)})$ for $i \in [\ell_{\mathsf{ek}}] \setminus \{k\}$. To simulate $\mathsf{ibe.sk}_k^{(j)}$, $\mathcal{B}$ proceeds as follows. If $\mathsf{pke.ek}^{(j)}[k] = 1 \oplus \mathsf{pke.ek}^{(\tilde{j})}[k]$, then $\mathcal{B}$ queries $\mathsf{id}^{(j)}$ to its IBE challenger and receives

   $$\mathsf{ibe.sk} \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}, \mathsf{id}^{(j)}).$$

   $\mathcal{B}$ then sets $\mathsf{ibe.sk}_k^{(j)} := \mathsf{ibe.sk}$. Otherwise, namely if $\mathsf{pke.ek}^{(j)}[k] = \mathsf{pke.ek}^{(\tilde{j})}[k]$, it computes $\mathsf{ibe.sk}_k^{(j)} \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}_{k,\mathsf{pke.ek}^{(j)}[k]}, \mathsf{id}^{(j)})$. Finally, $\mathcal{B}$ sends $d\mathcal{K}^{(j)} := (\{\mathsf{ibe.sk}_i^{(j)}\}_i, \mathsf{pke.ek}^{(j)}, \mathsf{pke}.d\mathcal{K}^{(j)})$ to $\mathcal{A}$ and adds $(\mathsf{id}^{(j)}, n, \mathsf{pke.vk}_n^{(j)}, \bot)$ to $L_{\mathcal{KG}}$.

   $O_{\mathcal{V}rfy}(\mathsf{id}, n, d\mathcal{K}')$**:** On input $(\mathsf{id}, n, d\mathcal{K}')$, $\mathcal{B}$ finds an entry $(\mathsf{id}, n, \mathsf{vk}, V)$ in $L_{\mathcal{KG}}$. If no such entry exists, it returns $\bot$. Otherwise, it parses $d\mathcal{K}' = (\{\mathsf{ibe.sk}_i'\}_i, \mathsf{pke.ek}, \mathsf{pke}.d\mathcal{K}')$ and $\mathsf{vk} = \mathsf{pke.vk}$. It computes $d := \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}, \mathsf{pke}.d\mathcal{K}')$ and returns d to $\mathcal{A}$. Finally, if $V = \bot$, it updates the entry to $(\mathsf{id}, n, \mathsf{vk}, d)$.
5. When $\mathcal{A}$ sends $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$ to the challenger, $\mathcal{B}$ defines $j^* \in [Q]$ as in Hyb_1. If $j^* \neq \tilde{j}$, then $\mathcal{B}$ aborts and outputs 0. Otherwise, $\mathcal{B}$ generates the challenge ciphertext ct^* as follows. It samples $\mathsf{R} \leftarrow \{0,1\}^{\ell_{\mathsf{rand}}}$ and computes $(\{\mathsf{lab}_{i,b}\}_{i\in[\ell_{\mathsf{ek}}],b\in\{0,1\}}, \widetilde{E}) \leftarrow \mathsf{Grbl}(1^\lambda, E[\mathsf{m}_0, \mathsf{R}])$. It then computes $\mathsf{ibe.ct}_{i,b}$ for all $(i,b) \in [\ell_{\mathsf{ek}}] \times \{0,1\} \setminus \{(k, 1\oplus\mathsf{pke.ek}^{(j^*)}[k])\}$ as in Equation (2). Next, $\mathcal{B}$ submits the target

identity $\mathsf{id}^* = \mathsf{id}^{(j^*)}$ and the message pair $(\mathsf{lab}_{k,\mathsf{pke.ek}^{(j^*)}[k]}, \mathsf{lab}_{k,1\oplus\mathsf{pke.ek}^{(j^*)}[k]})$ to its challenger. Then, the challenger returns

$$\mathsf{ibe.ct} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}, \mathsf{id}^{(j^*)}, \mathsf{lab}_{\overline{\mathsf{coin}}\oplus\mathsf{pke.ek}^{(j^*)}[k]}),$$

where $\overline{\mathsf{coin}} \in \{0,1\}$ is the challenge bit chosen by the IBE challenger. Then, $\mathcal{B}$ sets $\mathsf{ibe.ct}_{k,1\oplus\mathsf{pke.ek}^{(j^*)}[k]} := \mathsf{ibe.ct}$ and sends $\mathsf{ct}^* := (\{\mathsf{ibe.ct}_{i,b}\}_{i,b}, \widetilde{E})$ to $\mathcal{A}$.

6. The adversary $\mathcal{A}$ continues to make queries to $O_{\mathcal{KG}}(\cdot)$ and $O_{\mathcal{V}\mathit{rfy}}(\cdot,\cdot)$. $\mathcal{B}$ answers the queries in the same way as before the challenge query except that $O_{\mathcal{KG}}$ does not answer queries on the target identity id^*.
7. $\mathcal{A}$ finally outputs its guess. $\mathcal{B}$ outputs the same bit as its guess.

We first argue that $\mathcal{B}$ does not make any prohibited key query to its IBE challenger. The only case in which $\mathcal{B}$ queries its IBE challenger on an identity $\mathsf{id}^{(j)}$ is when $\mathsf{pke.ek}^{(j)}[k] = 1 \oplus \mathsf{pke.ek}^{(\tilde{j})}[k]$. Now suppose that $\mathsf{id}^{(j)} = \mathsf{id}^*$ and $j^* = \tilde{j}$. Since the same identity is always assigned the same randomness, the same identity induces the same encryption key. Hence, $\mathsf{pke.ek}^{(j)}[k] = \mathsf{pke.ek}^{(j^*)}[k] = \mathsf{pke.ek}^{(\tilde{j})}[k]$, and therefore $\mathsf{pke.ek}^{(j)}[k] = 1 \oplus \mathsf{pke.ek}^{(\tilde{j})}[k]$ never occurs. Thus, $\mathcal{B}$ never queries its IBE challenger on the target identity id^*.

We now analyze the advantage of $\mathcal{B}$. If $\overline{\mathsf{coin}} = 0$, then the challenge ciphertext component ibe.ct encrypts $\mathsf{lab}_{k,\mathsf{pke.ek}^{(j^*)}[k]}$, and the simulation is identical to $\mathsf{Hyb}_{2.(k+1)}$. If $\overline{\mathsf{coin}} = 1$, then the same component encrypts $\mathsf{lab}_{k,1\oplus\mathsf{pke.ek}^{(j^*)}[k]}$, and the simulation is identical to $\mathsf{Hyb}_{2.k}$.

Therefore, we have

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{ada\text{-}ind}}_{\mathsf{ABE},\mathcal{B}}(\lambda) &= 2\left|\Pr\left[\ \mathcal{B} \text{ outputs } \overline{\mathsf{coin}}\ \right] - \frac{1}{2}\right| \\
&= \left|\Pr\left[\ \mathcal{B} \text{ outputs } 1 \ |\overline{\mathsf{coin}} = 0\right] - \Pr\left[\ \mathcal{B} \text{ outputs } 1 \ |\overline{\mathsf{coin}} = 1\right]\right| \\
&= |\Pr[\mathsf{Hyb}_{2.k+1} = 1] - \Pr[\mathsf{Hyb}_{2.k} = 1]|
\end{aligned}$$

where the probabilities are taken over the randomness used in the respective games. Thus, $|\Pr[\mathsf{Hyb}_{2.k+1} = 1] - \Pr[\mathsf{Hyb}_{2.k} = 1]| = \mathsf{negl}(\lambda)$ by the adaptive security of ABE. This completes the proof of Lemma 9.4. □

Lemma 9.5. $|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]| = \mathsf{negl}(\lambda)$ *if* PKE-CR-SKL *is IND-KLA secure.*

Proof. We reduce the claim to the IND-KLA security of PKE-CR-SKL. Suppose that there exists an adversary $\mathcal{A}$ such that $|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]|$ is non-negligible. We construct an adversary $\mathcal{B}$ against the IND-KLA security of PKE-CR-SKL with the same advantage.

$\mathcal{B}(\mathsf{pke.ek})$**:** The adversary $\mathcal{B}$ works as follows.

1. The adversary $\mathcal{B}$ samples $\tilde{j} \leftarrow [Q]$ and sets $\mathsf{pke.ek}^{(\tilde{j})} := \mathsf{pke.ek}$. It request Q decryption keys from its IND-KLA challenger and receives $(\mathsf{pke.}\mathit{dk}_1, \ldots, \mathsf{pke.}\mathit{dk}_Q)$. It then generates $(\mathsf{ibe.pk}_{i,b}, \mathsf{ibe.msk}_{i,b}) \leftarrow \mathsf{IBE.Setup}(1^\lambda)$ for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$, and sends $\mathsf{pk} := \{\mathsf{ibe.pk}_{i,b}\}_{i,b}$ to $\mathcal{A}$.
2. The adversary $\mathcal{B}$ initializes the list $L_{\mathcal{KG}}$ as the empty set and simulates the following oracles for $\mathcal{A}$.

   $O_{\mathcal{KG}}(\mathsf{id}^{(j)})$**:** On the j-th query $\mathsf{id}^{(j)}$, where $j \in [Q]$, $\mathcal{B}$ finds entries of the form $(\mathsf{id}^{(j)}, 1, \mathsf{vk}_1, V_1), \ldots (\mathsf{id}^{(j)}, n-1, \mathsf{vk}_{n-1}, V_{n-1})$ in $L_{\mathcal{KG}}$.
   - If $j = \tilde{j}$ or $\mathsf{id}^{(j)} = \mathsf{id}^{(\tilde{j})}$, then $\mathcal{B}$ sets $\mathsf{pke.}\mathit{dk}_n^{(j)} := \mathsf{pke.}\mathit{dk}_n$ and $\mathsf{pke.vk}_n^{(j)} := \bot$. Here $n \le Q$.
   - Otherwise, $\mathcal{B}$ finds an entry $(\mathsf{id}^{(j)}, \rho^{(j)})$. If no such entry exists, it samples $\rho^{(j)}$ and stores $(\mathsf{id}^{(j)}, \rho^{(j)})$. It then generates $(\mathsf{pke.ek}^{(j)}, \mathsf{pke.msk}^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Setup}(1^\lambda; \rho^{(j)})$ and $(\mathsf{pke.}\mathit{dk}_n^{(j)}, \mathsf{pke.vk}_n^{(j)}) \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.}\mathcal{KG}(\mathsf{pke.msk}^{(j)})$
   - For every $i \in [\ell_{\mathsf{ek}}]$, $\mathcal{B}$ computes $\mathsf{ibe.sk}_i^{(j)} \leftarrow \mathsf{IBE.KG}(\mathsf{ibe.msk}_{i,\mathsf{pke.ek}^{(j)}[i]}, \mathsf{id}^{(j)})$.
   - It sends $\mathit{dk}^{(j)} := (\{\mathsf{ibe.sk}_i^{(j)}\}_i, \mathsf{pke.ek}^{(j)}, \mathsf{pke.}\mathit{dk}_n^{(j)})$ to $\mathcal{A}$ and adds $(\mathsf{id}^{(j)}, n, \mathsf{pke.vk}_n^{(j)}, \bot)$ to $L_{\mathcal{KG}}$.

$O_{\mathcal{V}rfy}(\mathsf{id}, n, d\mathcal{K}')$**:** On input $(\mathsf{id}, n, d\mathcal{K}')$, $\mathcal{B}$ finds an entry $(\mathsf{id}, n, \mathsf{pke.vk}_n^{\mathsf{id}}, V)$ in $L_{\mathcal{KG}}$. If there no such entry exists, it returns $\bot$. Otherwise, it parses $d\mathcal{K}' = (\{\mathsf{ibe.sk}'_i\}_i, \mathsf{pke.ek}', \mathsf{pke}.d\mathcal{K}')$. If $\mathsf{id} \neq \mathsf{id}^{(\tilde{j})}$, then $\mathcal{B}$ computes $d := \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}_n^{\mathsf{id}}, \mathsf{pke}.d\mathcal{K}')$ and returns d to $\mathcal{A}$. Otherwise, namely if $\mathsf{id} = \mathsf{id}^{(\tilde{j})}$ meaning $\mathsf{pke.vk}_n^{\mathsf{id}} = \bot$, $\mathcal{B}$ submits $(n, \mathsf{pke}.d\mathcal{K}')$ to its verification oracle. Then,

$$d := \mathsf{PKE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{pke.vk}_n, \mathsf{pke}.d\mathcal{K}')$$

is computed and returned to $\mathcal{B}$, where $\mathsf{pke.vk}_n$ is a verification key for $\mathsf{pke}.d\mathcal{K}_n$ given from its challenger. $\mathcal{B}$ then returns d to $\mathcal{A}$. Finally, if $V = \bot$, $\mathcal{B}$ updates the entry to $(\mathsf{id}, n, \mathsf{pke.vk}_n^{\mathsf{id}}, d)$.

3. When $\mathcal{A}$ sends $(\mathsf{id}^*, \mathsf{m}_0, \mathsf{m}_1)$ to the challenger, $\mathcal{B}$ first checks whether there exists an entry $(\mathsf{id}^*, n, \mathsf{pke.vk}_n, V_n)$ in $L_{\mathcal{KG}}$ such that $V_n = \bot$. If such an entry exists, $\mathcal{B}$ aborts and outputs 0. Otherwise, $\mathcal{B}$ defines $j^* \in [Q]$ as in Hyb_1. If $j^* \neq \tilde{j}$, then it aborts and outputs 0. Otherwise, $\mathcal{B}$ computes the challenge ciphertext ct^* as follows. It first submits $(\mathsf{m}_0, \mathsf{m}_1)$ to its challenger. Then, the challenger computes

$$\mathsf{pke.ct} \leftarrow \mathsf{PKE\text{-}CR\text{-}SKL.Enc}(\mathsf{pke.ek}, \mathsf{m}_{\overline{\mathsf{coin}}})$$

and returns it to $\mathcal{B}$, where $\overline{\mathsf{coin}} \in \{0,1\}$ is the challenge bit chosen by the challenger. The adversary $\mathcal{B}$ then computes $(\{\mathsf{lab}_i\}_{i\in[\ell_{\mathsf{ek}}]}, \widetilde{E}) \leftarrow \mathsf{Sim.GC}(1^\lambda, \mathsf{pke.ct})$ and, for every $i \in [\ell_{\mathsf{ek}}]$ and $b \in \{0,1\}$, computes $\mathsf{ibe.ct}_{i,b} \leftarrow \mathsf{IBE.Enc}(\mathsf{ibe.pk}_{i,b}, \mathsf{id}^*, \mathsf{lab}_i)$. It sends $\mathsf{ct}^* := (\{\mathsf{ibe.ct}_{i,b}\}_{i,b}, \widetilde{E})$ to $\mathcal{A}$.

4. The adversary $\mathcal{A}$ then continues to make queries to $O_{\mathcal{KG}}(\cdot)$ and $O_{\mathcal{V}rfy}(\cdot,\cdot)$. $\mathcal{B}$ answers the queries in the same way as before the challenge query except that $O_{\mathcal{KG}}$. does not answer queries on the target identity id^*.
5. $\mathcal{A}$ finally outputs its guess. $\mathcal{B}$ outputs the same bit as its guess.

We then have

$$\begin{aligned}\mathsf{Adv}^{\mathsf{ind\text{-}kla}}_{\mathsf{PKE\text{-}CR\text{-}SKL},\mathcal{B}}(\lambda) &= \left|\Pr\left[\ \mathcal{B} \text{ outputs } 1 \ |\overline{\mathsf{coin}} = 0\right] - \Pr\left[\ \mathcal{B} \text{ outputs } 1 \ |\overline{\mathsf{coin}} = 1\right]\right| \\ &= |\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]|\end{aligned}$$

where the probabilities are taken over the randomness used in the respective games. Thus, $|\Pr[\mathsf{Hyb}_4 = 1] - \Pr[\mathsf{Hyb}_5 = 1]| = \mathsf{negl}(\lambda)$ by the security of PKE-CR-SKL. This completes the proof of Lemma 9.5. □

This completes the proof of Theorem 9.3 for the case of adaptive security. □

Corollary 9.6. *There exists an adaptively IND-KLA secure IBE-CR-SKL scheme, assuming the polynomial hardness of the LWE assumption.*

This corollary immediately follows from Theorems 3.8, 3.15, 6.1 and 9.3.

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A Quantum Secure ABE for All Relations from LWE

We show there exists quantum selective-secure ABE for all relations computable in polynomial time based on the LWE assumption.

A.1 Preparation

Definition A.1 (Quantum-Accessible Pseudo-Random Function). *Let* $\{\mathsf{PRF}_k : \{0,1\}^{\ell_1} \to \{0,1\}^{\ell_2} \mid k \in \{0,1\}^\lambda\}$ *be a family of polynomially computable functions, where* ℓ_1 *and* ℓ_2 *are some polynomials of* λ*. We say that* PRF *is a quantum-accessible pseudo-random function (QPRF) family if for any QPT adversary* $\mathcal{A}$*, it holds that*

$$\mathsf{Adv}^{\mathsf{prf}}_{\mathcal{A}}(\lambda) := \left|\Pr\left[\mathcal{A}^{|\mathsf{PRF}_k(\cdot)\rangle}(1^\lambda) \leftarrow 1 \mid k \leftarrow \{0,1\}^\lambda\right] - \Pr\left[\mathcal{A}^{|\mathsf{R}(\cdot)\rangle}(1^\lambda) \leftarrow 1 \mid \mathsf{R} \leftarrow \mathcal{U}\right]\right| \leq \mathsf{negl}(\lambda)$$

where $\mathcal{U}$ *is the set of all functions from* $\{0,1\}^{\ell_1}$ *to* $\{0,1\}^{\ell_2}$*.*

Theorem A.2 ([Zha12]). *If there exists a OWF, there exists a QPRF.*

A.2 Proofs

We first define Key-Policy ABE for polynomial size circuits and briefly see that it can be used to instantiate ABE for any relations computable in polynomial time, even under quantum selective-security. Then, we prove that the Key-Policy ABE scheme for polynomial size circuits by Boneh et al. [BGG+14] with a light modification using QPRF satisfies quantum selective-security under the LWE assumption.

Key-Policy ABE for Circuits: Let $\mathcal{X}_\lambda = \{0,1\}^{n(\lambda)}$ and $\mathcal{Y}_\lambda$ be the set of all circuits with input space $\{0,1\}^{n(\lambda)}$ and size at most $s(\lambda)$, where n and s are some polynomials. Let R_λ be the following relation:

$$R_\lambda(x,y) = 0 \iff y(x) = 0$$

An ABE scheme for such $\{\mathcal{X}_\lambda\}_\lambda, \{\mathcal{Y}_\lambda\}_\lambda$, and $\{R_\lambda\}_\lambda$ is referred to as a Key-Policy ABE scheme for circuits.

Lemma A.3. *If there exists a quantum selective-secure Key-Policy ABE scheme for circuits, then there exists a quantum selective-secure ABE scheme for all relations computable in polynomial time.*

Proof. Let $\mathcal{X}_\lambda \subseteq \{0,1\}^n$, $\mathcal{Y}_\lambda \subseteq \{0,1\}^\ell$, and $R_\lambda : \mathcal{X}_\lambda \times \mathcal{Y}_\lambda \to \{0,1\}$, where n and ℓ are polynomials and R_λ is computable in polynomial time. We construct ABE scheme $\mathsf{ABE} = (\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$ with attribute spaces $\{\mathcal{X}_\lambda\}$ and $\{\mathcal{Y}_\lambda\}$ and relation $\{R_\lambda\}$, using Key-Policy ABE scheme for circuits $\mathsf{ABE}' = (\mathsf{Setup}, \mathsf{KG}', \mathsf{Enc}, \mathsf{Dec})$ with the following attribute spaces $\{\mathcal{X}'_\lambda\}$ and $\{\mathcal{Y}'_\lambda\}$.

- $\mathcal{X}'_\lambda = \{0,1\}^n$.
- Let $C_{\lambda,y}$ be the circuit such that $C_{\lambda,y}(x) = R_\lambda(x,y)$ for every $x \in \{0,1\}^n$ and $y \in \{0,1\}^\ell$, and let s be the maximum size of C_y. Note that s is a polynomial in λ. Then, $\mathcal{Y}'_\lambda$ is the set of all circuits with input length n and size at most s'.

The scheme is as follows: $\mathsf{KG}'(\mathsf{msk}, y, r) = \mathsf{KG}(\mathsf{msk}, C_{\lambda,y}, r)$. It is easy to see that the correctness of ABE follows from that of ABE'. For the quantum selective security of ABE', consider a reduction $\mathcal{R}$ to the quantum selective security of ABE.

Execution of $\mathcal{R}^{\mathcal{A}}$ in Experiment $\mathsf{Exp}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda, \mathsf{coin})$:

1. $\mathcal{R}$ receives challenge attribute $x^* \in \mathcal{X}_\lambda$ from $\mathcal{A}$ and forwards it to the challenger Ch.
2. For each oracle query (Y, Z) made by $\mathcal{A}$, $\mathcal{R}$ performs the following map on register Z initialized to $\left|0^{s(\lambda)}\right\rangle$:
$$|y\rangle_{\mathsf{Y}} |z\rangle_{\mathsf{Z}} \mapsto |y\rangle_{\mathsf{Y}} \left|z \oplus C_{\lambda,y}\right\rangle_{\mathsf{Z}}$$
3. Then, $\mathcal{R}$ queries the registers Y, Z to O_{qkg} followed by returning the registers Y, Z to $\mathcal{A}$.
4. When $\mathcal{A}$ sends $(\mathsf{m}_0, \mathsf{m}_1)$ to $\mathcal{R}$, $\mathcal{R}$ forwards it to Ch.
5. On receiving $\mathsf{ct}^\star \leftarrow \mathsf{Enc}(\mathsf{pk}, x^*, \mathsf{m}_{\mathsf{coin}})$ from Ch, $\mathcal{R}$ forwards it to $\mathcal{A}$.
6. Finally, when $\mathcal{A}$ outputs a guess coin', $\mathcal{R}$ sends coin' to Ch.

Since the view of $\mathcal{A}$ is identical to that in the quantum selective security experiment for scheme ABE', $\mathcal{R}$ ends up breaking the quantum selective security of ABE. □

Theorem A.4. *Assuming the polynomial hardness of the LWE problem, there exists a quantum selective-secure Key-Policy ABE scheme for circuits.*

Proof. We claim that the Key-Policy ABE scheme for circuits by Boneh et al. [BGG+14] based on LWE is quantum selective-secure. Actually, we will alter the key-generation algorithm of their scheme as follows: $\mathsf{KG}(\mathsf{msk}, y, k) = \mathsf{KG}'(\mathsf{msk}, y, \mathsf{PRF}_k(y))$ where KG' is the key-generation algorithm of their construction with explicit random coins $\mathsf{PRF}_k(y)$, and $\{\mathsf{PRF}_k\}_k$ is QPRF. This is a common technique utilized in quantum security proofs (See for Eg. [BZ13a, BZ13b]) that allows one to use a common random value for every term of a superposition. In the following discussion, whenever we discuss a hybrid titled "Game i" for some value i, it refers to the corresponding hybrid in Theorem 4.2 of [BGG+14]. Also, any indistinguishability claims between Game hybrids that are mentioned to be previously established, will refer to their work. Consider the following sequence of hybrids for the aforementioned ABE scheme ABE and a QPT adversary $\mathcal{A}$:

$\mathsf{Hyb}^{\mathsf{coin}}_0$**:** This is the same as the experiment $\mathsf{Exp}^{\mathsf{q\text{-}sel\text{-}ind}}_{\mathsf{ABE},\mathcal{A}}(1^\lambda, \mathsf{coin})$.

$\mathsf{Hyb}^{\mathsf{coin}}_1$**:** This is similar to $\mathsf{Hyb}^{\mathsf{coin}}_0$, except that the public-key pk is generated based on the challenge attribute x^*, as in the hybrid Game 1. It was shown that the hybrids Game 0 (the original experiment) and Game 1 are statistically indistinguishable. By the same argument, it follows that $\mathsf{Hyb}^{\mathsf{coin}}_0 \approx_s \mathsf{Hyb}^{\mathsf{coin}}_1$.

$\mathsf{Hyb}_2^{\mathsf{coin}}$**:** This is similar to $\mathsf{Hyb}_1^{\mathsf{coin}}$, except that the public-key is further modified as in Game 2. Unlike the change made in $\mathsf{Hyb}_1^{\mathsf{coin}}$ though, this change requires the key-queries to be answered differently as in Game 2. More specifically, the hybrid now has a punctured master secret-key that still allows it to simulate every key-query y such that $C_y(x^*) \neq \bot$. It was shown previously that Game 1 $\approx_s$ Game 2. Consider now the intermediate hybrids $\widetilde{\mathsf{Hyb}}_1^{\mathsf{coin}}, \widetilde{\mathsf{Hyb}}_2^{\mathsf{coin}}$ that are similar to $\mathsf{Hyb}_1^{\mathsf{coin}}, \mathsf{Hyb}_2^{\mathsf{coin}}$ respectively, except that all the superposition key-queries in these hybrids are responded using independent (true) randomness in every term of the superposition. We will now restate the following lemma by [BZ13a], which we will use to argue that $\widetilde{\mathsf{Hyb}}_1^{\mathsf{coin}} \approx_s \widetilde{\mathsf{Hyb}}_2^{\mathsf{coin}}$.

Lemma A.5. *[BZ13a] Let $\mathcal{Y}$ and $\mathcal{Z}$ be sets and for each $y \in \mathcal{Y}$, let D_y and D'_y be distributions on $\mathcal{Z}$ such that $\mathsf{SD}(D_y, D'_y) \leq \epsilon$. Let $O : \mathcal{Y} \to \mathcal{Z}$ and $O' : \mathcal{Y} \to \mathcal{Z}$ be functions such that $O(y)$ outputs $z \leftarrow D_y$ and $O'(y)$ outputs $z' \leftarrow D'_y$. Then, $O(y)$ and $O'(y)$ are ϵ'-statistically indistinguishable by quantum algorithms making q superposition oracle queries, such that $\epsilon' = \sqrt{8C_0q^3\epsilon}$ where C_0 is a constant.*

Recall that $\mathcal{Y}$ denotes the set of key-attributes. Let us fix a challenge ciphertext attribute x^* for the following discussion. For each $y \in \mathcal{Y}$ let the distributions $D_y^1[\mathsf{pk}], D_y^2[\mathsf{pk}]$ correspond to how a key for attribute y is sampled in the hybrids $\widetilde{\mathsf{Hyb}}_1^{\mathsf{coin}}, \widetilde{\mathsf{Hyb}}_2^{\mathsf{coin}}$ respectively, conditioned on the public key being pk. Note that for each $i \in [2]$, we consider $D_y^i[\mathsf{pk}]$ to also output the public-key pk along with the secret-key for y. We know that on average over pk, $D_y^1[\mathsf{pk}] \approx_s D_y^2[\mathsf{pk}]$ holds for all $y \in \mathcal{Y}$. It now follows from the above lemma that on average over pk, the analogously defined oracles $O_1[\mathsf{pk}], O_2[\mathsf{pk}]$ are statistically indistinguishable by algorithms making polynomially many superposition queries. Observe that with access to oracle $O_i[\mathsf{pk}]$ for each $i \in [2]$, the view of $\mathcal{A}$ in $\widetilde{\mathsf{Hyb}}_i^{\mathsf{coin}}$ (conditioned on the public-key being pk) can easily be recreated as the oracle outputs pk, which can then be used to compute the challenge ciphertext corresponding to coin. Consequently, it follows that the hybrids $\widetilde{\mathsf{Hyb}}_1^{\mathsf{coin}}$ and $\widetilde{\mathsf{Hyb}}_2^{\mathsf{coin}}$ are statistically indistinguishable. Since $\widetilde{\mathsf{Hyb}}_i^{\mathsf{coin}} \approx \mathsf{Hyb}_i^{\mathsf{coin}}$ holds for each $i \in [2]$ (by the quantum-security of PRF), we have that $\mathsf{Hyb}_1^{\mathsf{coin}} \approx \mathsf{Hyb}_2^{\mathsf{coin}}$.

$\mathsf{Hyb}_3^{\mathsf{coin}}$**:** This is similar to $\mathsf{Hyb}_2^{\mathsf{coin}}$, except that the challenge ciphertext is chosen uniformly at random, as in Game 3.

Observe that $\mathsf{Hyb}_3^0 \equiv \mathsf{Hyb}_3^1$. It was shown previously that Game 3 $\approx$ Game 2 by a reduction to LWE. Specifically, the reduction prepares the setup based on the LWE sample and the challenge ciphertext x^*, and plants the LWE challenge in the challenge ciphertext. It is easy to see that a similar reduction works in our case, thereby showing that $\mathsf{Hyb}_2^{\mathsf{coin}} \approx \mathsf{Hyb}_3^{\mathsf{coin}}$. Consequently, it follows that $\mathsf{Hyb}_0^0 \approx \mathsf{Hyb}_0^1$.

This completes the proof. □

B IND-CVA-CD Secure BB84 Based SKE-CD

To prove Theorem 3.19, we show how to transform IND-CD secure SKE-CD to IND-CVA-CD secure one.

Definition B.1 (IND-CD Security). *We define the security experiment $\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda, \mathsf{coin})$ in the same way as $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda, \mathsf{coin})$ except that the adversary $\mathcal{A}$ is allowed to get access to the verification oracle only once. We say that SKECD is IND-CD secure if for any QPT $\mathcal{A}$, it holds that*

$$\mathsf{Adv}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda, 0) = 1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda, 1) = 1\right]\right| \leq \mathsf{negl}(\lambda).$$

Bartusek and Khurana [BK23] showed the following theorem.

Theorem B.2 ([BK23]). *There exists an IND-CD secure BB84 based SKE-CD scheme assuming just an IND-CPA secure SKE scheme.*

We prove Theorem 3.19 by proving the following theorem.

Theorem B.3. *Let* $\mathsf{SKECD} = (\mathsf{KG}, \mathcal{Enc}, \mathcal{Dec}, \mathcal{Del}, \mathsf{Vrfy})$ *be a BB84-based SKE-CD scheme. If* SKECD *is IND-CD secure, then it is also IND-CVA-CD secure.*

Proof. Let $\mathcal{A}$ be a QPT adversary that attacks the IND-CVA-CD security of SKECD with q verification queries. Let $\mathtt{FV}_i$ be the event that the i-th verification query is the first verification query such that the answer for it is not $\bot$. Then, $\mathcal{A}$'s advantage can be described as follows.

$$\mathsf{Adv}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda) := \left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,0)=1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,1)=1\right]\right| \leq$$
$$\sum_{i\in[q]}\left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,0)=1\wedge \mathtt{FV}_i\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,1)=1\wedge \mathtt{FV}_i\right]\right| \tag{3}$$

To bound each term of Equation (3), we construct the following QPT adversary $\mathcal{B}_i$ that attacks the IND-CD security of SKECD using $\mathcal{A}$.

1. $\mathcal{B}_i$ initializes $\mathcal{A}$ with the security parameter 1^λ.
2. $\mathcal{A}$ makes queries to the encryption oracle throughout the experiment.

   $O_{\mathcal{Enc}}(\mathsf{m})$**:** When $\mathcal{A}$ makes an encryption query m, $\mathcal{B}_i$ forwards it to its own encryption oracle, and sends back the answer (vk, ct) from the encryption oracle to $\mathcal{A}$.
3. When $\mathcal{A}$ outputs $(\mathsf{m}_0,\mathsf{m}_1)\in\mathcal{M}^2$, $\mathcal{B}_i$ sends $(\mathsf{m}_0,\mathsf{m}_1)$ to its challenger. On receiving the challenge ciphertext ct^* from the challenger, $\mathcal{B}_i$ measures its classical part cla. This does not affect ct^*. $\mathcal{B}_i$ then forwards ct^* to $\mathcal{A}$.
4. Hereafter, $\mathcal{A}$ can get access to the following oracle.

   $O_{\mathsf{Vrfy}}(\mathsf{cert}_j)$**:** For the j-th query cert_j, if $j<i$, $\mathcal{B}_i$ returns $\bot$ to $\mathcal{A}$. if $j=i$, $\mathcal{B}_i$ queries cert_j to its verification oracle. If the response is $\bot$, $\mathcal{B}_i$ aborts. Otherwise if the answer is sk, $\mathcal{B}_i$ forwards sk to $\mathcal{A}$. $\mathcal{B}_i$ also computes θ from cla and sk, and sets $\mathsf{vk}'=(\theta,\mathsf{cert}_i)$. $\mathcal{B}_i$ checks whether $\mathsf{Vrfy}(\mathsf{vk}',\mathsf{cert}_j)=\bot$ holds for every $j<i$ (that is, whether the i-th query is the first query resulting in the answer other than $\bot$). If not, $\mathcal{B}_i$ aborts. Otherwise, $\mathcal{B}_i$ responds to the subsequent verification queries using vk'.
5. When $\mathcal{A}$ outputs $\mathsf{coin}'\in\{0,1\}$, $\mathcal{B}_i$ outputs coin'.

Let $\mathsf{vk}^*=(\theta,x)$ be the verification key corresponding to ct^*. For any string cert, if $\mathsf{Vrfy}(\mathsf{vk}^*,\mathsf{cert})=\top$, $\mathsf{Vrfy}(\mathsf{vk}^*,\cdot)$ and $\mathsf{Vrfy}(\mathsf{vk}',\cdot)$ for $\mathsf{vk}'=(\theta,\mathsf{cert})$ are functionally equivalent, and vk' can be used as an alternative verification key. This is because $\mathsf{Vrfy}(\mathsf{vk}^*,\mathsf{cert}')$ and $\mathsf{Vrfy}(\mathsf{vk}',\mathsf{cert}')$ respectively checks whether $\mathsf{cert}'[i]=x[i]$ and $\mathsf{cert}'[i]=\mathsf{cert}[i]$ holds or not for every $i\in[n]$ such that $\theta[i]=1$, and for such i, we have $x[i]=\mathsf{cert}[i]$ from the fact that $\mathsf{Vrfy}(\mathsf{vk}^*,\mathsf{cert})=\top$.

From the above, after the i-th verification query from $\mathcal{A}$ is responded, $\mathcal{B}_i$ can check whether its simulation of $\mathcal{A}$ so far has been successful or not. Moreover, if the simulation has failed, $\mathcal{B}_i$ aborts, and otherwise, $\mathcal{B}_i$ can successfully simulate the remaining steps for $\mathcal{A}$ using the alternative verification key $\mathsf{vk}'=(\theta,\mathsf{cert}_i)$. Then, we have $\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{B}_i}(1^\lambda,\mathsf{coin})=1\right] = \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{A}}(1^\lambda,\mathsf{coin})=1\wedge\mathtt{FV}_i\right]$. Since, SKECD satisfies IND-CD security, it holds that

$$\left|\Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{B}_i}(1^\lambda,0)=1\right] - \Pr\left[\mathsf{Exp}^{\mathsf{ind\text{-}cd}}_{\mathsf{SKECD},\mathcal{B}_i}(1^\lambda,1)=1\right]\right| = \mathsf{negl}(\lambda)$$

for every $i\in[q]$, which shows each term of Equation (3) is negligible. This completes the proof. □

C Construction of SKFE-CR-SKL with Key Testability

C.1 Construction

We construct an SKFE-CR-SKL scheme for the functionality $F : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$ with key testability $\mathsf{SKFE\text{-}CR\text{-}SKL} = \mathsf{SKFE\text{-}CR\text{-}SKL}.(\mathsf{Setup}, \mathcal{KG}, \mathsf{Enc}, \mathcal{Dec}, \mathcal{Vrfy})$ having the additional algorithms $\mathsf{SKFE\text{-}CR\text{-}SKL}.(\mathsf{CDec}, \mathsf{KeyTest})$, using the following building blocks.

- BB84-based SKE-CD scheme (Definition 3.18) $\mathsf{SKECD} = \mathsf{SKECD}.(\mathsf{KG}, \mathcal{Enc}, \mathcal{Dec}, \mathcal{Del}, \mathsf{Vrfy})$ having the classical decryption algorithm $\mathsf{SKECD.CDec}$.
- Classical SKFE scheme $\mathsf{SKFE} = \mathsf{SKFE}.(\mathsf{Setup}, \mathsf{KG}, \mathsf{Enc}, \mathsf{Dec})$ for the functionality $F : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$.
- OWF $f : \{0,1\}^\lambda \to \{0,1\}^{p(\lambda)}$ for some polynomial p.

The construction is as follows:

$\mathsf{SKE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$**:**

1. Generate $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$.
2. Generate $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE.Setup}(1^\lambda)$.
3. Output $\mathsf{msk} := (\mathsf{skecd.sk}, \mathsf{skfe.msk})$.

$\mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{KG}(\mathsf{msk}, y)$**:**

1. Parse $\mathsf{msk} = (\mathsf{skecd.sk}, \mathsf{skfe.msk})$.
2. Generate $\mathsf{skfe.sk}_y \leftarrow \mathsf{SKFE.KG}(\mathsf{skfe.msk}, y)$.
3. Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{Enc}(\mathsf{skecd.sk}, \mathsf{skfe.sk}_y)$. Recall that skecd.vk is of the form $(x,\theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, and skecd.$ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.
4. Generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $i \in [\ell_{\mathsf{ct}}]$ and $b \in \{0,1\}$. Set $T := t_{1,0} \| t_{1,1} \| \cdots \| t_{\ell_{\mathsf{ct}},0} \| t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i \in [\ell_{\mathsf{ct}}] \,:\, \theta[i]=1}$.
5. Prepare a register $\mathsf{S_i}$ that is initialized to $\left|0^\lambda\right\rangle_{\mathsf{S_i}}$ for every $i \in [\ell_{\mathsf{ct}}]$.
6. For every $i \in [\ell_{\mathsf{ct}}]$, apply the map
$$|u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i\rangle_{\mathsf{S_i}} \to |u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S_i}}$$
to the registers $\mathsf{SKECD.CT_i}$ and $\mathsf{S_i}$ and obtain the resulting state ρ_i.
7. Output $sk_y = (\rho_i)_{i \in [\ell_{\mathsf{ct}}]}$, $\mathsf{vk} = (x, \theta, S)$, and $\mathsf{tk} = T$.

$\mathsf{SKFE\text{-}CR\text{-}SKL.Enc}(\mathsf{msk}, x)$**:**

1. Parse $\mathsf{msk} = (\mathsf{skecd.sk}, \mathsf{skfe.msk})$.
2. Generate $\mathsf{skfe.ct} \leftarrow \mathsf{SKFE.Enc}(\mathsf{skfe.msk}, x)$.
3. Output $\mathsf{ct} := (\mathsf{skecd.sk}, \mathsf{skfe.ct})$.

$\mathsf{SKFE\text{-}CR\text{-}SKL.CDec}(\mathsf{sk}, \mathsf{ct})$**:**

1. Parse $\mathsf{ct} = (\mathsf{skecd.sk}, \mathsf{skfe.ct})$. Parse sk as a string over the registers $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$. Let $\widetilde{\mathsf{sk}}$ be the sub-string of sk on register $\mathsf{SKECD.CT}$.
2. Compute $\mathsf{skfe.sk} \leftarrow \mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{sk}})$.
3. Output $z \leftarrow \mathsf{SKFE.Dec}(\mathsf{skfe.sk}, \mathsf{skfe.ct})$.

$\mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{D}ec(s\mathcal{K}, \mathsf{ct})$:

1. Parse $(\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$. We denote the register holding ρ_i as $\mathsf{SKECD.CT}_i \otimes \mathsf{S}_i$ for every $i \in [\ell_{\mathsf{ct}}]$.
2. Prepare a register MSG of ℓ_{m} qubits that is initialized to $|0\cdots 0\rangle_{\mathsf{MSG}}$.
3. Apply the map
$$|u\rangle_{\mathsf{SKECD.CT}} |w\rangle_{\mathsf{MSG}} \to |u\rangle_{\mathsf{SKECD.CT}} \otimes |w \oplus \mathsf{SKFE\text{-}CR\text{-}SKL.CDec}(u, \mathsf{ct})\rangle_{\mathsf{MSG}}$$
to the registers $\mathsf{SKECD.CT} := \bigotimes_{i\in[\ell_{\mathsf{ct}}]} \mathsf{SKECD.CT}_i$ and MSG.
4. Measure MSG in the computational basis and output the result m'.

$\mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{V}rfy(\mathsf{vk}, s\mathcal{K})$:

1. Parse $\mathsf{vk} = (x, \theta, S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1})$ and $s\mathcal{K} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$ where ρ_i is a state on the registers $\mathsf{SKECD.CT}_i$ and S_i.
2. For every $i \in [\ell_{\mathsf{ct}}]$, measure ρ_i in the Hadamard basis to get outcomes c_i, d_i corresponding to the registers $\mathsf{SKECD.CT}_i$ and S_i respectively.
3. Output $\top$ if $x[i] = c_i \oplus d_i \cdot (s_{i,0} \oplus s_{i,1})$ holds for every $i \in [\ell_{\mathsf{ct}}]$ such that $\theta[i] = 1$. Otherwise, output $\bot$.

$\mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{tk}, \mathsf{sk})$:

1. Parse sk as a string over the registers $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$. Let u_i denote the value on $\mathsf{SKECD.CT}_i$ and v_i the value on S_i. Parse tk as $T = t_{1,0}\|t_{1,1}\|\cdots\|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$.
2. Let $\mathsf{Check}[t_{i,0}, t_{i_1}](u_i, v_i)$ be the deterministic algorithm that outputs 1 if $f(v_i) = t_{i,u_i}$ holds and 0 otherwise.
3. Output $\mathsf{Check}[t_{1,0}, t_{1,1}](u_1, v_1) \wedge \cdots \wedge \mathsf{Check}[t_{\ell_{\mathsf{ct}},0}, t_{\ell_{\mathsf{ct}},1}](u_{\ell_{\mathsf{ct}}}, v_{\ell_{\mathsf{ct}}})$.

C.2 Proof of Selective Single-Ciphertext KLA Security

Let $\mathcal{A}$ be an adversary for the selective single-ciphertext KLA security of the construction $\mathsf{SKFE\text{-}CR\text{-}SKL}$ that makes use of a BB84-based SKE-CD scheme SKECD. Consider the hybrid $\mathsf{Hyb}_j^{\mathsf{coin}}$ defined as follows:

$\mathsf{Hyb}_j^{\mathsf{coin}}$:

1. Initialized with 1^λ, $\mathcal{A}$ outputs (x_0^*, x_1^*). Sample $\mathsf{msk} \leftarrow \mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$.
2. $\mathcal{A}$ can get access to the following (stateful) oracles, where the list $L_{\mathcal{KG}}$ used by the oracles is initialized to an empty list:

   $O_{\mathcal{KG}}(y)$: Given y, it finds an entry of the form (y, vk, V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise it proceeds as follows:
   (i) If this is the k-th query for $k \le j$ and $F(x_0^*, y) \neq F(x_1^*, y)$, then compute $(s\mathcal{K}, \mathsf{vk}, \mathsf{tk}) \leftarrow \widetilde{\mathcal{KG}}(\mathsf{msk}, y)$ where $\widetilde{\mathcal{KG}}$ is defined below. Otherwise, compute $(s\mathcal{K}, \mathsf{vk}, \mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk}, y)$.
   (ii) It sends $s\mathcal{K}$ and tk to $\mathcal{A}$ and adds $(y, \mathsf{vk}, \bot)$ to $L_{\mathcal{KG}}$.

   $O_{\mathcal{V}rfy}(y, \widetilde{s\mathcal{K}})$: Given $(y, \widetilde{s\mathcal{K}})$, it finds an entry (y, vk, V) from $L_{\mathcal{KG}}$. (If there is no such entry, it returns $\bot$.) It then runs $d \leftarrow \mathcal{V}rfy(\mathsf{vk}, \widetilde{s\mathcal{K}})$ and returns d to $\mathcal{A}$. If $V = \top$, it updates the entry into (y, vk, d).

$\widetilde{\mathcal{KG}}(\mathsf{msk})$: Differences from $\mathcal{KG}$ are colored in red:

(a) Parse $\mathsf{msk} = (\mathsf{skecd.sk}, \mathsf{skfe.msk})$.
(b) Generate $r \leftarrow \{0,1\}^\lambda$.
(c) Generate $(\mathsf{skecd}.ct, \mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk}, r)$. $\mathsf{skecd.vk}$ is of the form $(x, \theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, and $\mathsf{skecd}.ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.

(d) Generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $i \in [\ell_{\mathsf{ct}}]$ and $b \in \{0,1\}$. Set $T := t_{1,0}\|t_{1,1}\|\cdots\|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1}$.

(e) Prepare a register S_i that is initialized to $\left|0^\lambda\right\rangle_{\mathsf{S}_\mathsf{i}}$ for every $i \in [\ell_{\mathsf{ct}}]$.

(f) For every $i \in [\ell_{\mathsf{ct}}]$, apply the map

$$|u_i\rangle_{\mathsf{SKECD.CT}_\mathsf{i}} \otimes |v_i\rangle_{\mathsf{S}_\mathsf{i}} \to |u_i\rangle_{\mathsf{SKECD.CT}_\mathsf{i}} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S}_\mathsf{i}}$$

to the registers $\mathsf{SKECD.CT}_\mathsf{i}$ and S_i and obtain the resulting state ρ_i.

(g) Output $s\mathcal{k}_y = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$, $\mathsf{vk} = (x,\theta,S)$, and $\mathsf{tk} = T$.

3. If there exists an entry (y,vk,V) in $L_{\mathcal{KG}}$ such that $F(x_0^*,y) \neq F(x_1^*,y)$ and $V = \bot$, output 0. Otherwise, generate $\mathsf{ct}^* \leftarrow \mathsf{Enc}(\mathsf{msk}, x^*_{\mathsf{coin}})$ and send ct^* to $\mathcal{A}$.
4. $\mathcal{A}$ continues to make queries to $O_{\mathcal{KG}}$. However, $\mathcal{A}$ is not allowed to send y such that $F(x_0^*,y) \neq F(x_1^*,y)$ to $O_{\mathcal{KG}}$.
5. $\mathcal{A}$ outputs a guess coin' for coin. Output coin'.

Let $\mathcal{A}$ make $q = \mathrm{poly}(\lambda)$ many queries to $O_{\mathcal{KG}}$ before the challenge phase. We will now prove the following lemma:

Lemma C.1. *$\forall j \in \{0,\ldots,q-1\}$ and $\mathsf{coin} \in \{0,1\}$: $\mathsf{Hyb}_j^{\mathsf{coin}} \approx_c \mathsf{Hyb}_{j+1}^{\mathsf{coin}}$.*

Proof. Suppose $\mathsf{Hyb}_j^{\mathsf{coin}} \not\approx \mathsf{Hyb}_{j+1}^{\mathsf{coin}}$. Let $\mathcal{D}$ be a corresponding distinguisher. We will construct a reduction $\mathcal{R}$ that breaks the IND-CVA-CD security of the BB84-based SKE-CD scheme SKECD. The execution of $\mathcal{R}^{\mathcal{D}}$ in the experiment $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{R}}(1^\lambda,b)$ proceeds as follows:

Execution of $\mathcal{R}^{\mathcal{D}}$ in $\mathsf{Exp}^{\mathsf{ind\text{-}cva\text{-}cd}}_{\mathsf{SKECD},\mathcal{R}}(1^\lambda,b)$:

1. The challenger $\mathcal{Ch}$ computes $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$.
2. $\mathcal{R}$ initializes $\mathcal{D}$ with 1^λ which outputs (x_0^*, x_1^*).
3. $\mathcal{R}$ samples $\mathsf{skfe.msk} \leftarrow \mathsf{SKFE.Setup}(1^\lambda)$. It initializes a list $L_{\mathcal{KG}}$ to an empty list.
4. $\mathcal{R}$ simulates the oracle $O_{\mathcal{KG}}$ for $\mathcal{D}$ as follows:

   $O_{\mathcal{KG}}(y)$: Given y, it finds an entry of the form (y,vk,V) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$. Otherwise, it proceeds as follows for the k-th query:

   (a) If $k \leq j$ and $F(x_0^*,y) \neq F(x_1^*,y)$, then compute $(s\mathcal{k},\mathsf{vk},\mathsf{tk})$ as in $\widetilde{\mathcal{KG}}$ except that the pair $(\mathsf{skecd}.ct,\mathsf{skecd.vk})$ is obtained as the output of $O_{\mathcal{E}nc}(r)$ instead. Send $s\mathcal{k}$ and tk to $\mathcal{A}$ and add $(y,\mathsf{vk},\bot)$ to $L_{\mathcal{KG}}$.

   (b) If $k = j+1$ and $F(x_0^*,y) \neq F(x_1^*,y)$, compute $\mathsf{skfe.sk}_y \leftarrow \mathsf{SKFE.KG}(\mathsf{skfe.msk},y)$. Then, send $(r^*,\mathsf{skfe.sk}_y)$ to $\mathcal{Ch}$ where r^* is a random value. On receiving $\mathsf{skecd.ct}^*$ from $\mathcal{Ch}$, compute $s\mathcal{k}$ and tk as in $\mathcal{KG}$, except that $\mathsf{skecd.ct}^*$ is used in place of $\mathsf{skecd.ct}$. Send $s\mathcal{k}$ and tk to $\mathcal{A}$ and add $(y,0^\lambda,\bot)$ to $L_{\mathcal{KG}}$.

   (c) If $k > j+1$ or $F(x_0^*,y) = F(x_1^*,y)$, then compute $(s\mathcal{k},\mathsf{vk},\mathsf{tk})$ as in $\mathcal{KG}$ except that the pair $(\mathsf{skecd}.ct,\mathsf{skecd.vk})$ is replaced with the output of $O_{\mathsf{Enc}}(\mathsf{skfe.sk}_y)$ instead. Send $s\mathcal{k}$ and tk to $\mathcal{A}$ and add $(y,\mathsf{vk},\bot)$ to $L_{\mathcal{KG}}$.

5. $\mathcal{R}$ simulates the oracle $O_{\mathcal{V}\mathit{rfy}}$ for $\mathcal{D}$ as follows:

   $O_{\mathcal{V}\mathit{rfy}}(y,\widetilde{s\mathcal{k}})$: Given $(y,\widetilde{s\mathcal{k}})$, it finds an entry (y,vk,V) from $L_{\mathcal{KG}}$ (If there is no such entry, it returns $\bot$.) It then proceeds as follows, if y corresponds to the k-th query made to $O_{\mathcal{KG}}$ and $k \neq j+1$:

   (a) Parse $\mathsf{vk} = (x,\theta,S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1})$ and $\widetilde{s\mathcal{k}} = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$.

   (b) For every $i \in [\ell_{\mathsf{ct}}]$, measure ρ_i in the Hadamard basis to get outcomes c_i, d_i corresponding to the registers $\mathsf{SKECD.CT}_\mathsf{i}$ and S_i respectively.

(c) Compute $\mathsf{cert}[i] = c_i \oplus d_i \cdot (s_{i,0} \oplus s_{i,1})$ for every $i \in [\ell_{\mathsf{ct}}]$.

(d) If $x[i] = \mathsf{cert}[i]$ holds for every $i \in [\ell_{\mathsf{ct}}] : \theta[i] = 1$, then update the entry to $(y, \mathsf{vk}, \top)$ and send $\top$ to $\mathcal{D}$. Else, send $\bot$.

If $k = j+1$, then it proceeds as follows:

(a) Compute $\mathsf{cert} = \mathsf{cert}[1] \| \ldots \| \mathsf{cert}[\ell_{\mathsf{ct}}]$ where each $\mathsf{cert}[i]$ is computed as in the previous case.

(b) Send cert to $\mathcal{CH}$. If $\mathcal{CH}$ returns $\mathsf{skecd.sk}$, send $\top$ to $\mathcal{D}$ and update the corresponding entry to $(y, \mathsf{vk}, \top)$. Else, output $\bot$.

6. $\mathcal{D}$ requests the challenge ciphertext. If there exists an entry (y, vk, V) in $L_{\mathcal{KG}}$ such that $F(x_0^*, y) \neq F(x_1^*, y)$ and $V = \bot$, output 0. Otherwise, compute and send $\mathsf{ct}^\star = (\mathsf{skecd.sk}, \mathsf{SKFE.Enc}(\mathsf{skfe.msk}, x^*_{\mathsf{coin}}))$ to $\mathcal{D}$.
7. $\mathcal{D}$ continues to make queries to $O_{\mathcal{KG}}$. However, $\mathcal{A}$ is not allowed to send y such that $F(x_0^*, y) \neq F(x_1^*, y)$ to $O_{\mathcal{KG}}$. Consequently, $\mathcal{R}$ simulates these queries as per Step 4. (c) above.
8. $\mathcal{D}$ outputs a guess coin' for coin which $\mathcal{R}$ forwards to $\mathcal{CH}$. $\mathcal{CH}$ outputs coin' as the output of the experiment.

We will first argue that when $b = 1$, the view of $\mathcal{D}$ is exactly the same as its view in the hybrid $\mathsf{Hyb}_j^{\mathsf{coin}}$. Notice that the for the first j queries, if $F(x_0^*, y) \neq F(x_1^*, y)$ for a key-query corresponding to y, then the decryption key is computed by querying the encryption oracle on a random plaintext. If this condition does not hold, then the key is computed by querying the encryption oracle on the corresponding SKFE key $\mathsf{skfe.sk}_y$. The hybrid Hyb_j on the other hand, directly computes these values, but there is no difference in the distribution of the output ciphertexts. A similar argument holds for the keys $\mathit{dk}_{j+2}, \ldots, \mathit{dk}_q$, which contain encryptions of the corresponding keys $\mathsf{skfe.sk}_y$. Note that if $F(x_0^*, y^*) = F(x_1^*, y^*)$, where y^* corresponds to the $j+1$-th query, then the hybrids $\mathsf{Hyb}_j^{\mathsf{coin}}$ and $\mathsf{Hyb}_{j+1}^{\mathsf{coin}}$ are identical. Hence, consider the case when $F(x_0^*, y^*) \neq F(x_1^*, y^*)$. In this case, if $b = 1$, the value encrypted as part of the key sk_{j+1} is $\mathsf{skfe.sk}_{y^*}$. This is the same as in $\mathsf{Hyb}_j^{\mathsf{coin}}$. As for the verification oracle queries, notice that they are answered similarly by the reduction and $\mathsf{Hyb}_j^{\mathsf{coin}}$ for all but the $j+1$-th key. For the $j+1$-th key, the reduction works differently in that it forwards the certificate cert to the verification oracle. However, the verification procedure of the BB84-based SKE-CD scheme checks the validity of the value cert in the same way as the reduction, so there is no difference. Finally, notice that when $b = 0$, the encrypted value is an independent and random value, similar to the hybrid $\mathsf{Hyb}_{j+1}^{\mathsf{coin}}$. Consequently, $\mathcal{R}$ breaks the IND-CVA-CD security of SKECD with non-negligible probability, a contradiction. □

Notice now that the hybrid $\mathsf{Hyb}_0^{\mathsf{coin}}$ is the same as experiment $\mathsf{Exp}^{\mathsf{sel\text{-}1ct\text{-}kla}}_{\mathsf{SKE\text{-}CR\text{-}SKL},\mathcal{A}}(1^\lambda, \mathsf{coin})$. From the previous lemma, we have that $\mathsf{Hyb}_0^{\mathsf{coin}} \approx_c \mathsf{Hyb}_q^{\mathsf{coin}}$. However, we have that $\mathsf{Hyb}_q^0 \approx_c \mathsf{Hyb}_q^1$ holds from the selective single-ciphertext security of the underlying SKFE scheme SKFE. This is because any key-query corresponding to y after the q-th query is such that $F(x_0^*, y) = F(x_1^*, y)$. For the first q queries, wherever this condition doesn't hold, the SKFE keys have been replaced with random values. Consequently, we have that $\mathsf{Hyb}_0^0 \approx_c \mathsf{Hyb}_0^1$, which completes the proof. □

C.3 Proof of Key-Testability

First, we will argue the correctness requirement. Recall that $\mathsf{SKFE\text{-}CR\text{-}SKL}.\mathcal{KG}$ applies the following map to a BB84 state $|x\rangle_\theta$, where $(x, \theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, for every $i \in [\ell_{\mathsf{ct}}]$:

$$|u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes |v_i\rangle_{\mathsf{S_i}} \to |u_i\rangle_{\mathsf{SKECD.CT_i}} \otimes \left|v_i \oplus s_{i,u_i}\right\rangle_{\mathsf{S_i}}$$

where $\mathsf{SKECD.CT_i}$ denotes the register holding the i-th qubit of $|x\rangle_\theta$ and $\mathsf{S_i}$ is a register initialized to $|0 \ldots 0\rangle_{\mathsf{S_i}}$.

Consider applying the algorithm $\mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}$ in superposition to the resulting state, i.e., performing the following map, where $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$, and KT is initialized to $|0\rangle$:

$$|u\rangle_{\mathsf{SKECD.CT}} |v\rangle_{\mathsf{S}} |\beta\rangle_{\mathsf{KT}} \to |u\rangle_{\mathsf{SKECD.CT}} |v\rangle_{\mathsf{S}} |\beta \oplus \mathsf{SKFE\text{-}CR\text{-}SKL.KeyTest}(\mathsf{tk}, u\|v)\rangle_{\mathsf{KT}}$$

where $\mathsf{tk} = T = t_{1,0}\|t_{1,1}\|\cdots\|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$. Recall that SKFE-CR-SKL.KeyTest outputs 1 if and only if $\mathsf{Check}[t_{i,0},t_{i,1}](u_i,v_i) = 1$ for every $i \in [\ell_{\mathsf{ct}}]$, where u_i, v_i denote the states of the registers $\mathsf{SKECD.CT}_i$ and S_i respectively. Recall that $\mathsf{Check}[t_{i,0},t_{i,1}](u_i,v_i)$ computes $f(v_i)$ and checks if it equals t_{i,u_i}. Since the construction chooses t_{i,u_i} such that $f(s_{i,u_i}) = t_{i,u_i}$, this check always passes. Consequently, measuring register KT always produces outcome 1.

We will now argue that the security requirement holds by showing the following reduction to the security of the OWF f. Let $\mathcal{A}$ be an adversary that breaks the key-testability of SKFE-CR-SKL. Consider a QPT reduction $\mathcal{R}$ that works as follows in the OWF security experiment:

Execution of $\mathcal{R}^{\mathcal{A}}$ in $\mathsf{Expt}^{\mathsf{owf}}_{f,\mathcal{R}}(1^\lambda)$:

1. The challenger chooses $s \leftarrow \{0,1\}^\lambda$ and sends $y^* = f(s)$ to $\mathcal{R}$.
2. $\mathcal{R}$ runs $\mathsf{SKFE\text{-}CR\text{-}SKL.Setup}(1^\lambda)$ and initializes $\mathcal{A}$ with input msk.
3. $\mathcal{R}$ picks a random $k^* \in [q]$.
4. $\mathcal{R}$ simulates the access of $\mathcal{A}$ to the oracle $O_{\mathcal{KG}}$ as follows, where the list $L_{\mathcal{KG}}$ is initialized to an empty list:

   $O_{\mathcal{KG}}(y)$**:** For the k-th query, do the following:
   (a) Given y, it finds an entry of the form (y,tk) from $L_{\mathcal{KG}}$. If there is such an entry, it returns $\bot$.
   (b) Otherwise, if $k \neq k^*$, it generates $(s\mathsf{k}_y,\mathsf{vk},\mathsf{tk}) \leftarrow \mathcal{KG}(\mathsf{msk},y)$, sends $(s\mathsf{k}_y,\mathsf{vk},\mathsf{tk})$ to $\mathcal{A}$, and adds (y,tk) to $L_{\mathcal{KG}}$.
   (c) Otherwise, if $k = k^*$, it generates $(s\mathsf{k}_y,\mathsf{vk},\mathsf{tk}) \leftarrow \widetilde{\mathcal{KG}}(\mathsf{msk},y)$ (differences from $\mathcal{KG}$ are colored in red). It then sends $(s\mathsf{k}_y,\mathsf{vk},\mathsf{tk})$ to $\mathcal{A}$ and adds (y,tk) to $L_{\mathcal{KG}}$.

   $\widetilde{\mathcal{KG}}(\mathsf{msk},y)$
   (a) Parse $\mathsf{msk} = (\mathsf{skecd.sk},\mathsf{skfe.msk})$.
   (b) Generate $\mathsf{skfe.sk}_y \leftarrow \mathsf{SKFE.KG}(\mathsf{skfe.msk},y)$.
   (c) Sample $(\mathsf{skecd}.ct,\mathsf{skecd.vk}) \leftarrow \mathsf{SKECD}.\mathcal{E}nc(\mathsf{skecd.sk},\mathsf{skfe.sk}_y)$. The value skecd.vk is of the form $(x,\theta) \in \{0,1\}^{\ell_{\mathsf{ct}}} \times \{0,1\}^{\ell_{\mathsf{ct}}}$, and skecd.$ct$ is of the form $|\psi_1\rangle_{\mathsf{SKECD.CT}_1} \otimes \cdots \otimes |\psi_{\ell_{\mathsf{ct}}}\rangle_{\mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}}$.
   (d) Choose an index $i^\star \in [\ell_{\mathsf{ct}}]$ such that $\theta[i^\star] = 0$. For every $i \in [\ell_{\mathsf{ct}}]$ such that $i \neq i^\star$, generate $s_{i,b} \leftarrow \{0,1\}^\lambda$ and compute $t_{i,b} \leftarrow f(s_{i,b})$ for every $b \in \{0,1\}$. For $i = i^\star$, set $t_{i^\star,1-x[i^\star]} = y^*$. Then, generate $s_{i^\star,x[i^\star]} \leftarrow \{0,1\}^\lambda$ and compute $t_{i^\star,x[i^\star]} = f(s_{i^\star,x[i^\star]})$. Set $T := t_{1,0}\|t_{1,1}\|\cdots\|t_{\ell_{\mathsf{ct}},0}\|t_{\ell_{\mathsf{ct}},1}$ and $S = \{s_{i,0} \oplus s_{i,1}\}_{i\in[\ell_{\mathsf{ct}}]\,:\,\theta[i]=1}$.
   (e) Prepare a register S_i that is initialized to $|0^\lambda\rangle_{\mathsf{S}_i}$ for every $i \in [\ell_{\mathsf{ct}}]$.
   (f) For every $i \in [\ell_{\mathsf{ct}}]$, apply the map

   $$|u_i\rangle_{\mathsf{SKECD.CT}_i} \otimes |v_i\rangle_{\mathsf{S}_i} \to |u_i\rangle_{\mathsf{SKECD.CT}_i} \otimes |v_i \oplus s_{i,u_i}\rangle_{\mathsf{S}_i}$$

   to the registers $\mathsf{SKECD.CT}_i$ and S_i and obtain the resulting state ρ_i.
   (g) Output $s\mathsf{k}_y = (\rho_i)_{i\in[\ell_{\mathsf{ct}}]}$, $\mathsf{vk} = (x,\theta,S)$, and $\mathsf{tk} = T$.
5. $\mathcal{A}$ sends a tuple of classical strings (y,sk,x^*) to $\mathcal{R}$. $\mathcal{R}$ outputs $\bot$ if there is no entry of the form (y,tk) in $L_{\mathcal{KG}}$ for some tk. Also, if $k \neq k^*$, $\mathcal{R}$ outputs $\bot$. Otherwise, $\mathcal{R}$ parses sk as a string over the registers $\mathsf{SKECD.CT} = \mathsf{SKECD.CT}_1 \otimes \cdots \otimes \mathsf{SKECD.CT}_{\ell_{\mathsf{ct}}}$ and $\mathsf{S} = \mathsf{S}_1 \otimes \cdots \otimes \mathsf{S}_{\ell_{\mathsf{ct}}}$ and measures the register $\mathsf{S}_{i^\star}$ to obtain an outcome $s_{i^\star}$. $\mathcal{R}$ then sends $s_{i^\star}$ to the challenger.

Notice that the view of $\mathcal{A}$ is the same as its view in the key-testability experiment, as only the value $t_{i^\star,1-x[i^\star]}$ is generated differently by forwarding the value y, but this value is distributed identically to the original value. Note that in both cases, $\mathcal{A}$ receives no information about a pre-image of $t_{i^\star,1-x[i^\star]}$. Now, $\mathcal{R}$ guesses the index k that $\mathcal{A}$ targets with probability $\frac{1}{q}$. By assumption, we have that $\mathsf{CDec}(\mathsf{sk},\mathsf{ct}) \neq F(x^*,y)$ where $\mathsf{ct} = \mathsf{Enc}(\mathsf{msk},x^*)$. The value sk can be

parsed as a string over the registers $\mathsf{SKECD.CT}$ and S. Let $\widetilde{\mathsf{sk}}$ be the sub-string of sk on the register $\mathsf{SKECD.CT}$. Recall that CDec invokes the algorithm $\mathsf{SKECD.CDec}$ on input $\widetilde{\mathsf{sk}}$. We will now recall a property of $\mathsf{SKECD.CDec}$ that was specified in Definition 3.18:

Let $(ct, \mathsf{vk} = (x, \theta)) \leftarrow \mathsf{SKECD.Enc}(\mathsf{skecd.sk}, \mathsf{skfe.sk}_y)$ where $\mathsf{skecd.sk} \leftarrow \mathsf{SKECD.KG}(1^\lambda)$. Now, let u be any arbitrary value such that $u[i] = x[i]$ for all $i : \theta[i] = 0$. Then, the following holds:

$$\Pr\left[\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, u) = \mathsf{skfe.sk}_y\right] \geq 1 - \mathsf{negl}(\lambda)$$

Consequently, if $\widetilde{\mathsf{sk}}$ is such that $\widetilde{\mathsf{sk}}[i] = x[i]$ for all $i : \theta[i] = 0$, where (x, θ) are specified by $\mathsf{vk}_{k^\star}$, then $\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{sk}})$ outputs the value $\mathsf{skfe.sk}_y$ with high probability. Since $\mathsf{CDec}(\mathsf{sk}, \mathsf{ct} = (\mathsf{skecd.sk}, \mathsf{skfe.ct} = \mathsf{SKFE.Enc}(\mathsf{skfe.msk}, x^*)))$ outputs $\mathsf{SKFE.Dec}(\mathsf{SKECD.CDec}(\mathsf{skecd.sk}, \widetilde{\mathsf{sk}}), \mathsf{skfe.ct})$, we have that it outputs x^* with high probability from the decryption correctness of SKFE. Therefore, it must be the case that there exists some index i for which $\widetilde{\mathsf{sk}} \neq x[i]$. With probability $\frac{1}{\ell_{\mathsf{ct}}}$, this happens to be the guessed value $i^\star$. In this case, $\mathcal{A}$ must output $s_{i^\star}$ on register S_i such that $f(s_{i^\star}) = t_{i^\star, 1-x[i^\star]} = y^*$. This concludes the proof. □

Since we have proved selective single-ciphertext security and key-testability, we can now state the following theorem:

Theorem C.2. *Assuming the existence of a BB84-based SKE-CD scheme and the existence of OWFs, there exists a selective single-ciphertext KLA secure SKFE-CR-SKL scheme satisfying the key-testability property.*